THOMAS J. NECHYBA

MICROECONOMICS

AN INTUITIVE APPROACH WITH CALCULUS

2ND EDITION

APPLICATIONS INCLUDED IN THIS TEXT

Chapter 2

Everyday Applications
Watching a Bad Movie
Renting Car vs. Taking Taxis
Dieting and Nutrition
Setting up a College Trust Fund

Business Applications
Pricing and Quantity Discounts
"Supersizing"
Frequent Flyer Perks
Choice in Calling Plans

Policy Applications
Food Stamp Programs and Other Types of Subsidies
Public Housing and Housing Subsidies
Taxing Goods vs. Lump Sum Taxes
Public Schools and Private School Vouchers
Tax Deductions and Tax Credits

Chapter 3

Everyday Applications
Robots as Labor-Saving Products
Investing for Retirement
Different Interest Rates for Borrowing and Lending

Business Applications
Present Value of Winning Lottery Tickets
Picking Savings Accounts
Compound Interest over the Long Run
Pricing Government Bonds
Buying Houses with Annuities
A Trick for Calculating the Value of Annuities

Policy Applications
Wage Taxes and Budget Constraints
Proportional vs. Progressive Wage Taxes
Social Security (or Payroll) Taxes
AFDC vs. a Negative Income Tax
The Earned Income Tax Credit
Three Proposals to Deal with the Social Security Shortfall

Chapter 4

Everyday Applications
Rating Movies on a Numerical Scale
Did 9/11 Change Tastes?
Tastes of a Cocaine Addict

Business Applications
Tastes for Cars and Product Characteristics
Investor Tastes over Risk and Return

Policy Applications
Ideology and Preferences of Politicians
Subsistence Levels of Consumption

Chapter 5

Everyday Applications
Personality and Tastes for Current and Future Consumption
Thinking about Old Age
Tastes for Paperclips
Inferring Tastes for "Mozartkugeln"
Syllabi-Induced Tastes over Exam Grades

Chapter 6

Everyday Applications
Price Fluctuations in the Housing Market
Different Interest rates for Borrowing and Lending

Business Applications
Quantity Discounts and Optimal Choices
Retail Industry Lobbying for Daylight Savings Time

Policy Applications
Food Stamp vs. Food subsidies
Gasoline Taxes Tax Rebates
AFDC and Work Disincentives
Cost of Living Adjustments of Social Security Benefits

Chapter 7

Everyday Applications
Housing Price Fluctuations
Turkey and Thanksgiving

Business Applications
Sam's Club and the Marginal Consumer
Are Gucci Products Giffen Goods

Policy Applications
Tax Deductibility and Tax Credits
Substitution Effects and Social Security Cost of Living Adjustments
Fuel Efficiency, Gasoline Consumption and Gas Prices
Public Housing and Housing Subsidies

Chapter 8

Business Applications
Merchandise Exchange Policies

Policy Applications
Savings Behavior and Tax Policy
The Negative Income Tax
The Earned Income Tax Credit
Advising Congress on Savings Subsidies and Substitution Effects
International Trade and Child Labor
Subsidizing Savings and Taxing Borrowing
Tax Revenues and the Laffer Curve

Chapter 9

Everyday Applications
Backward-Bending Labor Supply

Business Applications
Price Discounts, Substitutes, and Complements
Good Apples vs. Bad Apples

Policy Applications
Tax and Retirement Policy

Chapter 10

Everyday Applications
Teacher Pay and Professional Basketball Salaries
Ordering Appetizers
To Trade or Not To Trade Pizza Coupons
To Join or Not To Join the Local Pool
To Take or Not to Take the Bus

Business Applications
Pricing at Disneyland
Negotiating an Endorsement Deal and a Bribe

Policy Applications
Distortionary Taxes
Price Subsidies
Taxing Interest on Savings
International Trade and Child Labor
Efficient Land Taxes

Chapter 11

Everyday Applications
Workers as Producers of Consumption
Studying for an Exam

Business Applications
Optimal Response to Labor Regulations
Technological Change in Production

Policy Applications
Politicians as Producers of Good Feelings
Determining Optimal Class Size

Chapter 12

Everyday Applications
To Study or to Sleep?
Fast Food Restaurants and Grease

Business Applications
Fast Food Restaurants and Grease
Investing in Smokestack Filters under Cap and Trade

Policy Applications
Investing in Smokestack Filters under Cap and Trade
Taxes on Firms

Chapter 13

Business Applications
Switching Technologies
Switching Technologies: Short-run vs. Long Run
Fixed Amount of Land for Oil Drilling
Minimum Wage Labor Subsidy
Business Taxes

Policy Applications
Fixed Amount of Land for Oil Drilling
Minimum Wage Labor Subsidy
Business Taxes

Chapter 14

Everyday Applications
Fast Food Restaurants and Grease

Business Applications
Fast Food Restaurants and Grease
Brand Names and Franchise Fees
"Economic Rent" and Profiting from Entrepreneurial Skill
Capital Gains Taxes
Using License Fees to Make Positive Profit
Business Taxes
Minimum Wage Labor Subsidy

Policy Applications
Capital Gains Taxes
Using License Fees to Make Positive Profit
Business Taxes
Minimum Wage Labor Subsidy
School Vouchers and the Private School Market
Public School Teacher Salaries, Class Size, and Private School Markets
Pollution Taxes on Output

Chapter 15

Everyday Applications
Labor-Saving Technologies

Business Applications
Disneyland Pricing Revisited
License Fees and Surplus without Income Effects

Policy Applications
License Fees and Surplus without Income Effects
Redistribution of Income without Income Effects
Deadweight Loss from Subsidy of Mortgage Interest
Markets, Social Planners, and Pollution
Anti-Price Gauging Laws

Chapter 16

Everyday Applications
Children, Parents, Baby Booms, and Baby Busts
Parents, Children, and the Degree of Substitutability across Time

Business Applications
Valuing Land in Equilibrium
Hiring an Assistant

Policy Applications
Distortionary Taxes and Redistribution
Distortionary Taxes in General Equilibrium
The Laffer Curve in General Equilibrium

Chapter 17

Everyday Applications
Gambling on Sporting Events
Teenage Sex and Birth Control
Gambling with Different Beliefs
Venice and Regret

Business Applications
Choosing a Mafia for Insurance
Diversifying Risk along the Business Cycle
Diversifying Risk along the Business Cycle in General Equilibrium
Local vs. National Insurance

Policy Applications
More Police or More Jails?
More Police or More Teachers? Enforcement vs. Education

Chapter 18

Everyday Applications
Scalping College Basketball Tickets

Business Applications
Minimum Wage Laws
Usury Laws
Subsidizing Corn Through Price Floors

Policy Applications
Minimum Wage Laws
Usury Laws
Subsidizing Corn Through Price Floors
Rent Control
NYC Taxi Cab Medallions
Kidney Markets
Oil Shocks and Gasoline Prices

Chapter 19

Business Applications
City Wage Taxes
Land Use Policies
Price Floors for Corn: Is It a Tax or a Subsidy?

Policy Applications
City Wage Taxes
Land Use Policies
Price Floors for Corn: Is It a Tax or a Subsidy?
Rent Control: Is It a Tax or a Subsidy?
Incidence of U.S. Social Security Taxes
Mortgage Interest Deductibility, Land Values, and the Equilibrium Rate of Return
The Split-Rate Property Tax

Chapter 20

Everyday Applications
Quality of Life Indexes
Trade, Migration, and the Price of a Haircut
Compensating Wage Differentials and Increased Worker Safety
Adaptive Expectations and Oil Price Instability

Business Applications
Compensating Wage Differentials and Increased Worker Safety
Adaptive Expectations and Oil Price Instability
The Risks of Short-Selling
Pricing Call and Put Options
General Equilibrium Effects of a Property Tax
Local Differences in Property Taxes

Policy Applications
General Equilibrium Effects of a Property Tax
Local Differences in Property Taxes
U.S. Immigration Policy
Trade Barriers against "Unfair" Competition

Chapter 21

Everyday Applications
Children's Toys and Gucci Products

Business Applications
Fishing in the Commons
The Externality when Fishing in the Commons
Network Externalities and the Battle between Microsoft and Apple
Pollution that Increases Firm Costs—The Market Outcome

Policy Applications
The Externality when Fishing in the Commons
Pollution that Increases Firm Costs—Barney's Solution
Pollution that Increases Firm Costs—Policy Solutions
Social Norms and Private Actions

Chapter 22

Everyday Applications
Non-Random "Selection" is Everywhere

Business Applications
Competitive Provision of Health Insurance

Policy Applications
Expanding Health Insurance Coverage
Moral Hazard versus Adverse Selection in Health Care Reform
Statistical Profiling in Random Car Searches

continued

APPLICATIONS INCLUDED IN THIS TEXT *continued*

Chapter 23

EVERYDAY APPLICATIONS
Diamonds Are a Girl's Best Friend

BUSINESS APPLICATIONS
Diamonds Are a Girl's Best Friend
Monopoly Pricing in Health Insurance Markets
Second Degree Price Discrimination in Health Insurance Markets
Labor Unions Exercising Market Power
Monopsony: A Single Buyer in the Labor Market
Taxing Monopoly Output
Two Natural Monopolies: Microsoft vs. Utility Companies

POLICY APPLICATIONS
Monopoly Pricing in Health Insurance Markets
Second Degree Price Discrimination in Health Insurance Markets
Labor Unions Exercising Market Power
Monopsony: A Single Buyer in the Labor Market
Taxing Monopoly Output
Two Natural Monopolies: Microsoft versus Utility Companies
Some Possible "Remedies" to the Monopoly Problem
Pollution and Monopolies
Regulating Market Power in the Commons

Chapter 24

EVERYDAY APPLICATIONS
Splitting the Pot
Another Way to Split the Pot
Real-World Mixed Strategies
Burning Money, Iterated Dominance, and the Battle of the Sexes
Bargaining Over a Fixed Amount
Auctions
Education as a Signal
To Fight or Not to Fight
Reporting a Crime

BUSINESS APPLICATIONS
Bargaining Over a Fixed Amount
Auctions
Monopoly and Price Discrimination
Carrots and Sticks: Efficiency Wages and the Threat of Firing Workers
Education as a Signal
To Fight or Not to Fight

POLICY APPLICATIONS
Negotiating with Pirates, Terrorists (and Children)
Education as a signal
To Fight or Not to Fight
Reporting a Crime
Some Prisoner's Dilemmas

Chapter 25

BUSINESS APPLICATIONS
Entrepreneurial Skill and Market Conditions
Quitting Time: When to Exit a Declining Industry
Financing a Strategic Investment under Quantity Competition
Financing a Strategic Investment under Price Competition
Deal or No Deal—Acquisitions of Up-Start Firms by Incumbents
Production Quotas under Cartel Agreements

POLICY APPLICATIONS
Production Quotas under Cartel Agreements
Mergers, Cartels, and Antitrust Enforcement
Subsidizing an Oligopoly
Government Grants and Cities as Cartels

Chapter 26

EVERYDAY APPLICATIONS
Cities and Land Values

BUSINESS APPLICATIONS
Mergers and Antitrust Policy in Related Product Markets
Advertising as Quality Signal
Price Leadership in Differentiated Product Markets
The Evolution of the Fashion Industry
Deterring Entry of Another Car Company
The Software Industry

POLICY APPLICATIONS
Mergers and Antitrust Policy in Related Product Markets
Lobbying for Car Import Taxes
The Software Industry
To Tax or Not to Tax Advertising

Chapter 27

EVERYDAY APPLICATIONS
Sandwiches, Chess Clubs, Movie Theaters, and Fireworks
Competitive Local Public and Club Good Production
Raising Money for a Streetlight through a "Subscription Campaign"

BUSINESS APPLICATIONS
Competitive Local Public and Club Good Production
Raising Money for a Streetlight through a "Subscription Campaign"
The Marketing Challenge for Social Entrepreneurs

POLICY APPLICATIONS
Competitive Local Public and Club Good Production
Demand for Charities and Tax Deductibility
Do Antipoverty Efforts Provide a Public Good?
Distortionary Taxes and National Security
Social Norms and Private Actions
The Pork Barrel Commons
Local and National Public Goods as Pork Barrel Projects

Chapter 28

EVERYDAY APPLICATIONS
Why Vote?
"Winner-Take-All" Elections and the U.S. Electoral College
To Enter or Not to Enter the Race
Citizen Candidates

BUSINESS APPLICATIONS
Voting with Points

POLICY APPLICATIONS
Voting with Points
Interest Groups, Transactions Costs, and Vote Buying
Political Coalitions and Public School Finance Policy
Government Competition, Leviathan, and Benevolence
The Pork Barrel Commons and the "Law of 1/N"

Chapter 29

EVERYDAY APPLICATIONS
Reference Points, Happiness, and Envy
Extreme Altruism and Normative Economics
Framing the Options: Praying while Smoking, and Fighting Pandemics
Teaser Rates on Credit Cards and Mortgages
Impatient versus Present-Biased Students
Endowment Effects and Housing Markets

BUSINESS APPLICATIONS
Teaser Rates on Credit Cards and Mortgages
Endowment Effects and Housing Markets
The Equity Premium Puzzle
Increased Liquidity, Procrastination, and National Savings

POLICY APPLICATIONS
Impatient versus Present-Biased Students
Increased Liquidity, Procrastination, and National Savings
First- and Second-Best Rawlsian Income Redistribution
Confirmation Bias, Politics, Research, and Last-Minute Studying
Smoothing the Business Cycle versus Fostering Economic Growth: Psychology Meets Normative Macroeconomics

MICROECONOMICS

AN INTUITIVE APPROACH WITH CALCULUS

2ND EDITION

MICROECONOMICS

AN INTUITIVE APPROACH WITH CALCULUS

2ND EDITION

Thomas J. Nechyba
Duke University

CENGAGE Learning®

Australia • Brazil • Mexico • Singapore • United Kingdom • United States

CENGAGE Learning™

Microeconomics: An Intuitive Approach with Calculus, 2e
Thomas J. Nechyba
Print and Video Graph Presentations: Deborah Antkoviak and John Gross of econweb™

Vice President, General Manager, Social Science & Qualitative Business: Erin Joyner

Product Director: Michael Worls

Associate Product Manager: Tara K. Singer

Associate Content Developer: Leah G. Wuchnick

Marketing Manager: Katie Jergens

Senior Content Project Manager: Colleen A. Farmer

Manufacturing Planner: Kevin Kluck

Production Service: Lumina Datamatics, Inc.

Sr. Art Director: Michelle Kunkler

Internal & Cover Designer: Red Hangar Design, LLC

Cover and Internal Design Image: © Subbotina Anna/Shutterstock

Intellectual Property Analyst: Jen Nonemacher

Project Manager: Sarah Swainwald

Library of Congress Control Number: **2015948039**
ISBN: **978-1-305-65046-6**

Cengage Learning
20 Channel Center Street
Boston, MA 02210
USA

Cengage Learning is a leading provider of customized learning solutions with employees residing in nearly 40 different countries and sales in more than 125 countries around the world. Find your local representative at **www.cengage.com.**

Cengage Learning products are represented in Canada by Nelson Education, Ltd.

To learn more about Cengage Learning Solutions, visit **www.cengage.com**

Purchase any of our products at your local college store or at our preferred online store **www.cengagebrain.com**

Printed in Canada
Print Number: 01 Print Year: 2015

BRIEF CONTENTS

CONTENTS

PREFACE

TO STUDENTS

As a student, I often felt both alienated and insulted by textbooks: alienated because they seemed to make no attempt to speak *to* rather than *at* me, insulted because they seemed to talk *down* to me by giving me lots of "visuals" (like pictures of monkeys—seriously) to keep me awake and by feeding me endless definitions to memorize—all while never acknowledging the obvious conceptual limits of what was being presented.

I have therefore tried to write a book that is a little different and that I think I might have liked to use when I was a student. Some have commented that you might not like it because it doesn't lend itself to memorizing definitions for exams. Others find it strange that I address you so directly throughout much of the book and that I occasionally even admit that this or that assumption we make is in many ways "silly."

I don't actually have anything against monkeys or definitions or assumptions that seem "silly," but my experience with students over the years tells me that you do not mind being challenged a bit and actually enjoy being part of a conversation rather than committing one to memory. The modern world has few rewards for people who are really good at memorizing but offers much to those who can conceptualize ideas and integrate them with one another. Economics offers a path to practice this—and it does so in a way that can be exciting and interesting, interesting enough to not actually require monkey pictures even if it is sometimes frustrating to get through some of the details.

I will say more about much of this in Chapter 1—so I'll try to avoid repeating myself here and instead just offer a few points on how best to use this text:

1. You may want to review parts of **Chapter 0** (which is not included in the print version of the text, but available through MindTap) to review some basics before proceeding to Chapter 2.
2. Attempt the **within-chapter exercises** as you read—and check your answers with those in the **Study Guide** or those included directly into the MindTap eReader. (My students, on whom I conducted quasi-controlled experiments during the initial drafting of this text, have done considerably better on exams when using within-chapter exercises and solutions.)
3. When skimming chapters, make sure the points emphasized in the margins of the text make sense—and focus on sections of the chapter where they don't.
4. Graphs with purple bars at the bottom can be unpacked directly within the MindTap eReader, and almost all graphs are available to view as animated and narrated videos that can be accessed through MindTap. While some of the video animations are long, you can skip ahead and use chapter markers to locate the part of the video you are most interested in.
5. Look for interesting applications in **end-of-chapter exercises**, but know that some of these are designed to be challenging. Don't get frustrated if they don't make sense at first. It helps to work with others to solve these (assuming your instructor allows this). All odd numbered exercises are accompanied by the † symbol that denotes exercises with solutions provided in the **Study Guide**. These solutions can also be accessed directly within the MindTap eReader.
6. While you will often feel like you are getting lost in details within chapters, the **Introductions** (to the Parts as well as the Chapters) and the **Conclusions** (in each chapter) attempt to keep an eye on the big picture. Don't skip them!

7. The book has an extensive **Glossary** and **Index** but develops definitions within a narrative rather than pulling them out within the text. Use the Glossary to remind yourself of the meaning of terms and the Index to find where the associated concepts are discussed in detail. But resist the temptation to memorize too much. The terms aren't as important as the concepts.
8. As the book goes to press, I am developing an online course consisting of short lecture modules as well as links to animations and worked out solutions. This course will be accessible through the EcoTeach Center at Duke (http://econ.duke.edu/ecoteach).

TO INSTRUCTORS

When I was first asked to teach microeconomics, I was surprised to learn that the course had been one of the least popular in my department. It was unclear what the goals of the course were—and without such clarity at the outset, students had come to view the course as a disjointed mess of graphs and math with little real-world relevance and no sense of what value it could add. As I came to define what goals I would like *my* course to develop, I had trouble finding a text that would help my students aim toward these goals without over-emphasizing just one or two to the exclusion of others. So we largely de-emphasized textbooks—but something was working: the course had suddenly become one of the most popular in the department!

I have therefore attempted to build a framework around the five primary goals that I believe any microeconomics course should accomplish:

1. It should present microeconomics not as a collection of unrelated models but ***as a way of looking at the world***. People respond to incentives because they try to do the best they can given their circumstances. That's microeconomics in a nutshell—and everything—*everything*—flows from it.
2. It should persuade that microeconomics does not just change the way *we think* about the world—it also tells us a lot about ***how and why the world works*** (and sometimes doesn't work).
3. It should not only get us to think more clearly about economics but also ***to think more clearly in general***—without relying on memorization. Such *conceptual thinking skills* are the very skills that are most sought after and most rewarded in the modern world.
4. It should directly confront the fact that few of us can move from memorizing to conceptual thinking without ***applying concepts directly***, but different students learn differently, and instructors need the ***flexibility*** to target material to *their* students' needs.
5. Finally, it should provide students with a ***roadmap for further studies***—a sense of what the most compelling next courses might be given *their* interests.

I am thus trying to provide a flexible framework that keeps us rooted in a *way of thinking* while developing a *coherent overview* to help us better understand the world around us. Half the text builds up to the most fundamental result in all of economics—that self-interested individuals will—*under certain conditions and without intending to*—give rise to a spontaneous order that has great benefits for society. But the second half probes these "certain conditions" and develops insights into how firms, governments, and civil society can contribute to human welfare when markets by themselves "fail." Future courses can then be seen as sub-fields that come to terms with these "certain conditions."

While the material in the full text is more than enough for a two-semester sequence, the text offers a **variety of flexible paths for a one-semester course**. In each chapter, you can emphasize an intuitive A part or link it to a more mathematical B part; and, while the last part of the text relies heavily on game theory, the underlying narrative can also be developed through a non-game theoretic approach. Substantive paths include some focused on *theory*, others focused on *policy*, and yet others focused on *business*, with all paths including core material as well as optional topics. Throughout, the models build in complexity, with applications woven into the narrative (rather than being relegated to side-boxes). They are then further developed in an extensive array of exercises that get students—not me or you—to apply concepts to *Everyday*, *Business*, and *Policy* settings.

For more details on how you might use the various parts of the text and its accompanying tools, I hope you will have a look at the **Instructor's Manual** that I have written to go along with the text.

While the student study guide includes answers to all odd numbered end-of-chapter exercises (in addition to answers to within-chapter exercises), answers to all end-of-chapter exercises are available to instructors. Here are just a few examples of how you might weave through the book depending on your focus. (These are depicted in more detail in the instructor edition of the text.)

1. **Traditional Theory Emphasis:**
   Ch. 1–23 (with Ch. 3, 8, the latter sections of 9 and 13 optional) plus
   Ch. 29–30 optional

2. **Theory Emphasis with Game Theory:**
   Ch. 1–18 (with 3, 8, the latter sections of 9, 13, and 18 optional) plus
   Ch. 23–27 (with 28 through 30 optional)

3. **Business Focus:**
   Ch. 1–18 (with Ch. 3, 8, 16, the latter sections of 9, 13, and 18 optional) plus
   Ch. 23–26

4. **Policy Focus:**
   Ch. 1–15 (with Ch. 3, 8, and the latter sections of 9 and 13 optional), plus
   Ch. 18–23, 28–30 (with Ch. 24–27 optional depending on level of game theory usage)

If you have suggestions for improvements to the text for the next edition, please feel free to contact me directly through my Duke email address.

ACKNOWLEDGMENTS

In addition to all those acknowledged in the first edition to the text, I would like to thank the Cengage team that has shepherded the second edition to its conclusion: Mike Worls, product director, Tara Singer, associate product manager, Colleen Farmer, senior content project manager, Michelle Kunkler, senior art director, and Kevin Kluck, manufacturing planner. Most visible to me has been the extraordinary efforts of Leah Wuchnick who has been the content developer and the answer to my many silent prayers leading up to undertaking this project. I could not have hoped for a better partner. Of course, Leah worked with a dedicated team on her end, and their efforts—while less visible to me—are equally appreciated.

Throughout the past few years, I have used the text in my microeconomics course at Duke, and so I must thank the many hundreds of students who, through their use of the materials and their many comments, have continued to improve the content. In addition, I have heard from students and instructors from around the world, and I hope some of them will see at least some of their comments reflected in this edition. Above all, I appreciate the encouragement from many who have found the text helpful and have made the effort to reach out.

Finally, it turns out that I have a full time job and a busy family life—and a project like this could not be seen to completion without the patient understanding of many who fill in the inevitable gaps that arise when one's focus shifts a bit. To all my colleagues, particularly those in the EcoTeach center and those in the Social Science Research Institute at Duke, thank you for not making me feel too guilty for the "blocked out" times on my calendar. And above all—to Stacy, Ellie, Jenny, and Katie—thank you always for being there.

Thomas J. Nechyba
Durham, NC

FURTHER ACKNOWLEDGMENTS

The author and editorial team at Cengage Learning would also like to acknowledge the feedback and assistance of many instructors who reviewed this text in various drafts and participated in several focus groups concerning its development. Their time and comments have been invaluable in crafting the final product.

J. Ulyses Balderas, *Sam Houston State University*

Klaus G. Becker, *Texas Tech University*

Allen Bellas, *Metropolitan State College of Denver*

Tibor Besedes, *Louisiana State University*

Maharukh Bhiladwalla, *Rutgers University*

Volodymyr Bilotkach, *University of California Irvine*

Benjamin F. Blair, *Mississippi State University*

Victor Brajer, *California State University Fullerton*

Nancy Brooks, *University of Vermont*

James Cardon, *Brigham Young University*

Kalyan Chakraborty, *Emporia State University*

Basanta Chaudhuri, *Rutgers University*

Ben Collier, *Northwest Missouri State University*

Stephen Davis, *Southwest Minnesota State University*

Mary Deily, *Lehigh University*

Wayne Edwards, *University of Alaska*

Adem Y. Elveren, *University of Utah*

Robert M. Feinberg, *American University*

Rhona Free, *Eastern Connecticut State University*

Jaqueline Geoghegan, *Clark University*

Dipak Ghosh, *Emporia State University*

Rajeev K. Goel, *Illinois State University*

Tiffani A. Gottschall, *Washington and Jefferson College*

Chiara Gratton- Lavoie, *California State University Fullerton*

Tom Gresik, *University of Notre Dame*

Philip Grossman, *St. Cloud State University*

Jim Halteman, *Wheaton College*

Vladimir Hlasny, *Michigan State University*

Gary Hoover, *University of Alabama*

Joseph Hughes, *Rutgers University*

Michael Keane, *Yale University*

Farida Khan, *University of Wisconsin Parkside*

Byung-Cheol Kim, *Georgia Institute of Technology*

Felix Kwan, *Maryville University*

Ross LaRoe, *Denison University*

Marc Law, *University of Vermont*

Sang H. Lee, *Southeastern Louisiana University*

Robert J. Lemke, *Lake Forest College*

Anthony M. Marino, *University of Southern California*

Douglas J. Miller, *University of Missouri*

Joshua B. Miller, *University of Minnesota Twin Cities*

Ranganath Murthy, *Bucknell University*

Kathryn Nantz, *Fairfield University*

Tara Natarjan, *Saint Michael's College*

Ronald Nate, *Brigham Young University*

Catherine Norman, *Johns Hopkins University*

Terry Olson, *Truman State University*

Mete Ozcan, *Brooklyn College CUNY*

Ebru Isil Ozturk, *University of Wisconsin*

Silve Parviainen, *University of Illinois at Urbana-Champaign*

Brian Peterson, *Central College*

Jonas Prager, *New York University*

James Prieger *Pepperdine University School of Public Policy*

Salim Rashid, *University of Illinois at Urbana-Champaign*

Tyler R. Ross, *University of Washington Seattle*

Jeremy Sandford, *University of Wisconsin*

Jonathan Sandy, *University of San Diego*

Mustafa Sawani, *Truman State University*

Kwang Soo Cheong, *Johns Hopkins University*

Charles Steele, *Hillsdale College*

Vasant Sukhatme, *Macalester College*

Jeffrey Sundberg, *Lake Forest College*

Jose Vasquez-Cognet, *University of Illinois at Urbana-Champaign*

Richard Vogel, *Farmingdale State College*

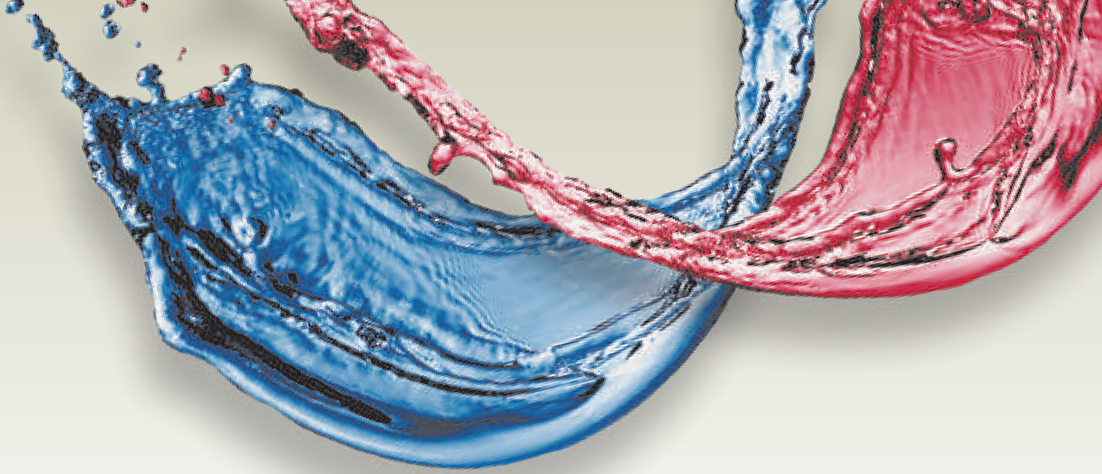
Eleanor von Ende, *Texas Tech University*

Rob Wassmer, *California State University Sacramento*

Tetsuji Yamada, *Rutgers University*

Ben Young, *University of Missouri Kansas City*

Sourushe Zandvakili, *University of Cincinnati*



CHAPTER 0

Foundational Preliminaries

In principle, this textbook presumes no prior knowledge of economics and thus essentially starts from scratch. However, it is undoubtedly true that an introductory level economics course (or some equivalent high school AP course) will give you a bit of a head start in the sense that some concepts—particularly toward the middle of the text—will already be familiar. The mathematical B-parts of the chapters presume a basic comfort level with algebra and some selected pre-calculus topics as well as the ability to take a derivative of a single-variable function. The primary higher-level mathematical technique utilized in the text—optimization methods using partial derivatives—is developed in the text without any presumption that you have seen this before—although again, you have a bit of a head start if you have taken multi-variable calculus (which is typically covered in the third course of a college calculus sequence).

This preliminary chapter is meant to get everyone "onto the same page" to minimize the gap between those with and those without prior economics and advanced math preparation. Particularly, in part A of the chapter we will review some basics of graphing the kinds of objects that we will graph in the A-parts of the book chapters and will apply them to some of the ideas you will have seen in an introductory course. In part B of this chapter, we will analogously cover the mathematical basics that you encounter in the B-parts of the book chapters—up to but not including the multi-variable calculus concepts that are directly developed within the text.

0A SOME GRAPHICAL PRELIMINARIES

The graphs in the A-parts of the text are ways of representing underlying mathematical objects that could in principle be treated without graphs (as developed in the B-parts). The great advantage of graphs is that they often more clearly illustrate economic intuitions that are sometimes not immediately apparent in a more traditional mathematical development of the same concepts. It is for this reason that economists rely so much on graphs and often illustrate their ideas graphically even if they have developed them in much more general form through mathematical models. Even the seasoned professional economist will find himself or herself drawing the kinds of pictures we develop in the A-parts of chapters to help make sense of answers that they get from solving mathematical equations. Math helps us to make sure that we are not playing tricks on ourselves when we just draw graphs, but the graphs often help us understand what is really going on underneath the math.

0A.1 Graphing Points and Sets

In this section, we'll review some basic issues related to graphing points and sets, and we will use modified versions of actual graphs from the text to illustrate the underlying concepts. Most of what we draw throughout the text is represented in two-dimensional graphs, with an x-axis and a y-axis that measure different types of variables like price and quantity. But these two-dimensional graphs are usually just "slices" of higher dimensional graphs—slices in which we hold something else fixed in order to be able to illustrate what we are focusing on in just two dimensions. We will sometimes remind ourselves of this and will find that keeping this in mind can help us visualize more complex ideas while not overly straining our graphing skills.

0A.1.1 Points and Sets in One Dimension Let's consider first the idea of a *point.* In one dimension, we usually think of a point as represented by a real number drawn from the number line that ranges from minus infinity to infinity. Such a real number might be an *integer* by which we simply mean a whole number (like 2 or 5 or 12); or it might be a *rational* number that can be represented as a fraction that divides one integer by another (like 4/3 or 2/9); or it might be an *irrational* number that lies on the real number line but cannot be represented as a fraction of one integer divided by another (such as $\sqrt{2}$ or π).[1]

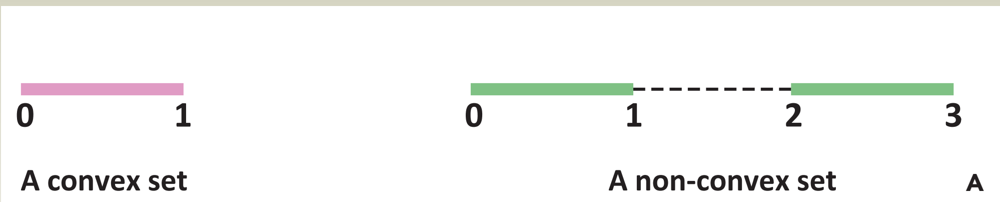
A set is convex if the line connecting any two points (in the set) is itself fully contained in the set.

A *set* of points in one dimension is then some subset of the real number line. The magenta interval from zero to 1 in Graph 0.1, denoted as [0,1], is an example of a set of points in one dimension. There are lots of properties of sets that we could define, but the one that appears frequently in the textbook is that of *convexity.* It is a pretty straightforward concept: A set of points is said to be *convex* if the straight line that connects any two points in the set is fully contained in the set. For instance, consider the magenta set [0,1]: No matter which two points in that set we pick, the line that connects the points lies on the real number line and within the magenta set [0,1]. Thus, [0,1] is a convex set.

A set that is not convex is called non-convex.

But now consider the green set that is made up of the segment [0,1] *and* [2,3], a set we would call the *union* of [0,1] and [2,3], denoted as $[0,1] \cup [2,3]$. We have now created a set that has a "hole" in it because it starts at 0, stops at 1 and then starts again at 2. This implies that we can pick a pair of numbers like 0.5 and 2.5 from the set $[0,1] \cup [2,3]$—with the line that connects the two points not fully contained in the set $[0,1] \cup [2,3]$. This is because the line that connects 0.5 to 2.5 contains the dashed interval [1,2] that represents the "hole" in our set. As a result, we would say that the set $[0,1] \cup [2,3]$ is not a convex set, and we will therefore call it a *non-convex set.* Notice that, in order for a set to be non-convex, all we have to do is find one pair of points from the set such that the line connecting those points lies at least partially outside the set. The fact that we can also find pairs of points whose connecting line lies fully in the set is irrelevant—for the set to be convex, *all* lines connecting *any* pair of points from the set must fully lie within the set.

GRAPH 0.1 Sets of One-Dimensional Points



[1]Irrational numbers can also be defined as numbers with non-ending and non-repeating decimals. As it turns out, almost all real numbers are irrational even though we often use rational numbers as approximations.

Consider the set [0,1), which includes the point 0, all the points in between 0 and 1 but not the point 1. Is this set convex? What about the union of this set with the point 1 ? What about the union of this set with the point 1.1?

EXERCISE **0A.1**

Is the set of all rational numbers a convex set? What about the set of all non-integers? Or the set of all irrational numbers?

EXERCISE **0A.2**

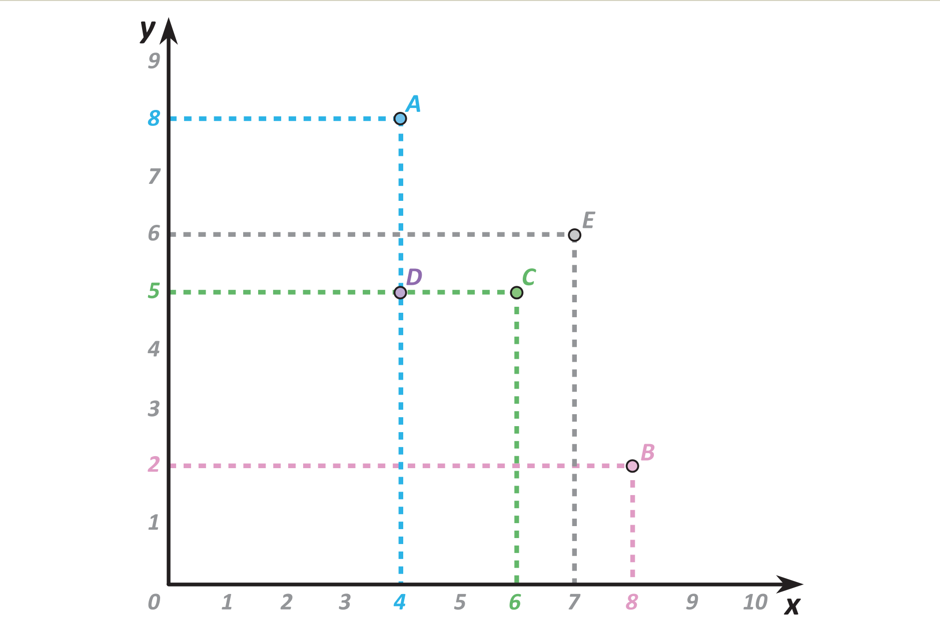
0A.1.2 Points and Sets in Two Dimensions If we understand the idea of points and sets in one dimension, it is a small step to understanding the same concepts in two dimensions. While a point in one dimension is *one* number from the real number line, a point in two dimensions is a pair of *two* numbers from the real number line. For instance, the point (4,8) is a point that measures 4 units on the horizontal *x*-axis of a graph and 8 units on the vertical *y*-axis of a graph, a point such as the point denoted by *A* in Graph 0.2.

Describe points *B*, *C*, *D* and *E* in Graph 0.2 as a pair of real numbers.

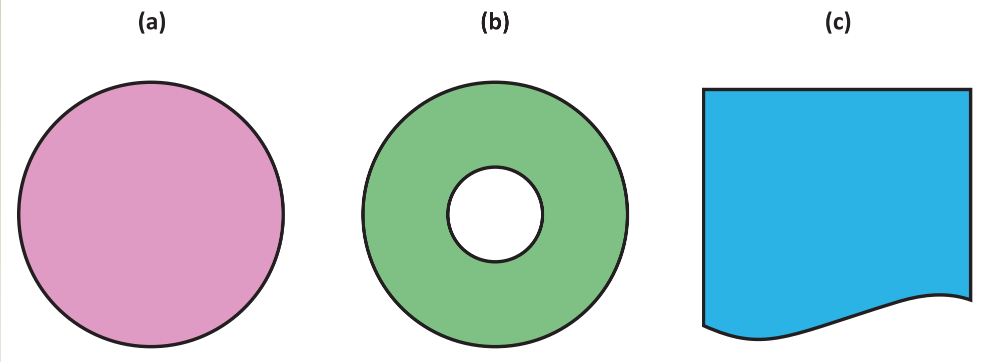
EXERCISE **0A.3**

Of course there is no more reason to restrict ourselves to points made up of integer values when we are in two dimensions than when we are in one dimension. The green dashed line segment between points *C* and *D* in Graph 0.2, for instance, contains points that have non-integer *x*-values (ranging from 4 to 6) while holding the *y*-value fixed at 5, and the dashed blue line segment between *A* and *D* contains points that have non-integer *y*-values (ranging from 5 to 8) while holding the *x*-value constant at 4. You can then quickly see that every "point" in the graph is in fact a *point* described by a pair of numbers, one referring to the *x*-dimension and another referring to the *y*-dimension.

GRAPH 0.2 Points in Two Dimensions



GRAPH 0.3 Two-Dimensional Sets



Once we understand the concept of a point in two dimensions, we can then again define sets of points as simply collections of points. For instance, we could define the set $\{A, B, C, D, E\}$ as the set of the five points graphed in Graph 0.2. And a set will again be *convex* if and only if all line segments that connect any two points in the set are fully contained within the set.[2]

EXERCISE 0A.4 Which of the sets in Graph 0.3 is/are not convex?

One particular type of set is the set of points that form a *line segment*. You will remember from an algebra class that any line in a two-dimensional graph can be represented by the equation

$$y = b + mx \tag{0.1}$$

Equations of lines with vertical intercept b and slope m can be written in the form $y = b + mx$.

where b represents the vertical intercept of the line and m represents its slope. Graph 0.4, for instance, contains a blue line segment that lies on the line with vertical intercept of 20 (at point B) and slope of –2. The line on which the blue segment lies is thus represented by the equation $y = 20 - 2x$. From the graph, we can calculate the value of the slope as the "rise" over the "run" as we move from one point on the line to another. Going from C to F, for instance, we go down by 2 (to E), which is a negative "rise" of 2, and 1 to the "right" (from E), which is a positive "run" of 1. This gives us

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{-2}{1} = -2. \tag{0.2}$$

EXERCISE 0A.5 Can you use points A and B to arrive at the same value for the slope? What about the points D and F or the points D and C?

The line segment in Graph 0.4 is itself a set of points—that is, the set of points with positive x- and y-values that lie on the line described by the equation $y = 20 - 2x$. It is in some sense no different than a line segment on the real number line except that it is placed into the two-dimensional plane. So long as we are simply defining the points on a line segment as a set, we are thus defining an object no different from what we defined in one dimension, with the idea of

[2]We will re-visit Graph 0.2 in the context of tastes and consumer goods in Chapter 4, where the x-axis will denote the number of pants in a consumer's basket, and the y-axis will denote the number of shirts.

convexity no different than it was in the previous section. But we could also use the line to define the boundary of a set that truly is two- rather than one-dimensional. The shaded area below the blue line in Graph 0.4, for instance, is the set of all points (with positive x- and y-values) that lie below the line—or the set of all points such that

$$y < 20 - 2x,\ x > 0 \text{ and } y > 0. \tag{0.3}$$

Such a set is again said to be *convex* if and only if you can take *any* two points within the set, connect them with a straight line segment and have the *whole* line segment contained within the set.[3]

Is the shaded set in Graph 0.4 a convex set?

EXERCISE 0A.6

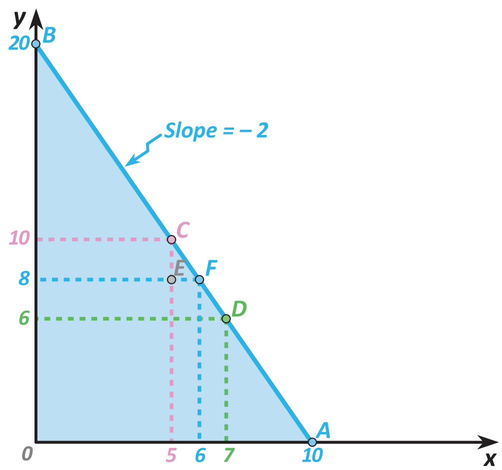
Suppose the blue line in Graph 0.4 had a kink in it. This kink could point "inward" (i.e., toward to origin) or "outward" (i.e., away from the origin). For which of these would the shaded area underneath the kinked line become a non-convex set?

EXERCISE 0A.7

0A.1.3 Three-Dimensions and Two-Dimensional "Slices" When we graph in two dimensions, we are necessarily limiting ourselves to considering how two—and only two—variables are related to one another. The line in Graph 0.4, for instance, tells us how the variable y on the vertical axis is related to the variable x on the horizontal axis (through the equation $y = 20 - 2x$). But we will see that it is sometimes useful to think about how two variables are related to a third variable. In such cases, we would need a three-dimensional graph to illustrate the full set of relationships that are of interest to us. An example of such a three-dimensional graph is given in Graph 0.5 where, in addition to the x and y variables, we now also have a z variable on the axis pointing toward you.

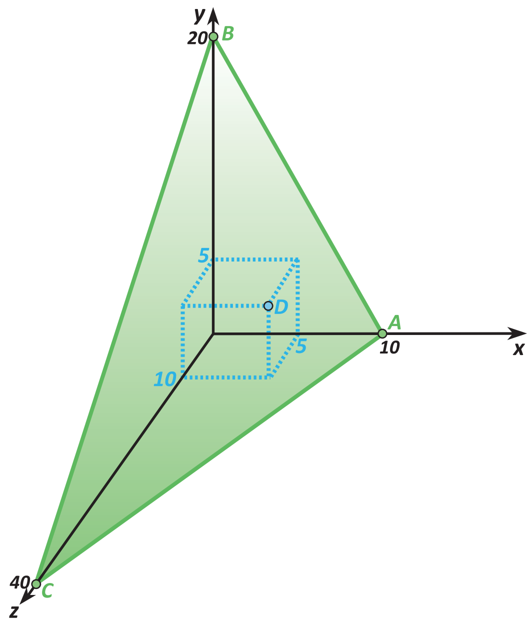
Just as a point in one dimension is described by just a number on the real number line, and a point in two dimensions is described by a pair of numbers indicating the x- and the y-value, a point in three dimensions is described by a triple of numbers—indicating the x-, y-, and z-values of the point. In Graph 0.5, for instance, the point $D = (5, 5, 10)$ is a point with x- and y-values

GRAPH 0.4 Lines and Sets Formed by Lines



[3] In Chapter 2, with "pants" on the x-axis and "shirts" on the y-axis, sets of this type will denote the set of affordable baskets of pants and shirts (given some money budget and given prices for pants and shirts).

GRAPH 0.5 Three-Dimensional Points and Sets



of 5 and a z-value of 10. It lies on the shaded plane that connects the points A, B, and C, which occur on the three axes of the graph, and this plane is itself a set of points in three dimensions.

The equation for the shaded plane is given by

$$4x + 2y + z = 40. \tag{0.4}$$

The intercepts on each axis can then easily be identified by setting the values of variables on the other axes to zero. For instance, the z-intercept at C occurs where y and x are both set to zero—which, according to equation (0.4), implies $z = 40$.

EXERCISE 0A.8 Check to see that the other intercepts (at *B* and *A*) are correctly labeled based on equation (0.4).

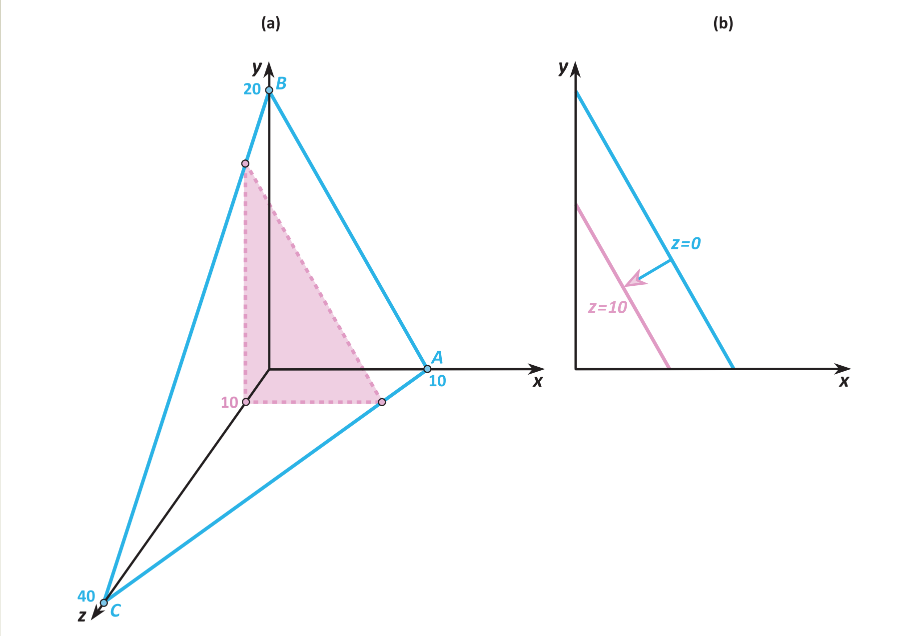
EXERCISE 0A.9 Is the plane in Graph 0.5 a convex set?

When one variable in a three-dimensional graph is held fixed, the resulting "slice" can be graphed in two dimensions.

Suppose then that x, y, and z are three economic variables, and that we know that, for the application at hand, $z = 0$. In that case, we could simply look at the two dimensions of the graph that hold z fixed at zero—which reduces the graph to just the x- and y-axes. We would then be left with a line that has y intercept of 20 and x intercept of 10—that is, a line described by the equation $y = 20 - 2x$ just as the one we graphed in Graph 0.4. The two-dimensional graph in Graph 0.4 is therefore a two-dimensional "slice" of the three-dimensional graph in Graph 0.5 when z is held fixed at 0.

0A.1.4 Switching between "Slices" as "Shifts in Curves" But z might not be appropriately set to zero —that is, the relevant "slice" for our purposes might lie at some level of z greater than zero. In Chapter 2, for instance, we will have an application where x represents "pants," y represents "shirts" and z represents "socks." The three-dimensional plane in Graph 0.5

GRAPH 0.6 Slices of Three-Dimensional Graphs



represents the set of baskets of pants, shirts, and socks that are affordable to a consumer. But suppose the consumer has already decided to buy 10 socks and we wanted to illustrate her remaining affordable baskets of shirts and pants. The appropriate "slice" of the graph is then one that occurs at $z = 10$—which is graphed as the magenta slice in panel (a) of Graph 0.6.

EXERCISE 0A.10

Given the equation (0.4) that describes the three-dimensional plane, what is the equation that describes the magenta line segment, which intersects with the plane in panel (a)?

In the two-dimensional x/y graph in panel (b), this change in the z variable from 0 to 10 then appears as a *shift* in the line segment. This is a general insight which will apply to many of the graphs we draw in the text: If a two-dimensional graph arises from some third variable being held fixed, changing that third variable is equivalent to shifting to a different "slice" of an underlying three-dimensional graph—with that shift appearing as a shift in a line or curve when projected onto the x/y graph.

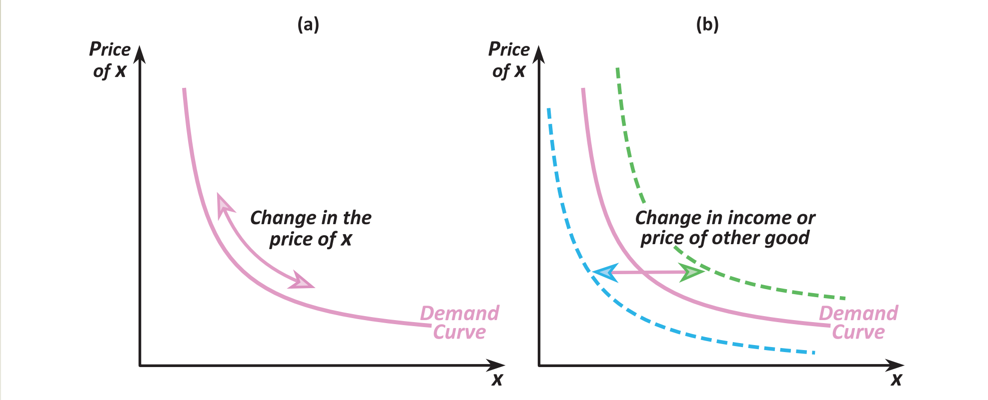
When a two-dimensional graph arises from holding a third variable fixed, then changing the value of that third variable will cause a shift in the two-dimensional graph.

0A.2 Demand and Supply Curves

If you have ever taken a class in economics, you have no doubt graphed demand and supply curves. We will not actually graph these fully within the text until we have done some work on what's behind these curves, because it is only then that we really understand fully what these curves actually mean. But, since you are probably already familiar with some basics of demand and supply curves, we can use them here to illustrate some of the reasons behind the *movements along curves* and *shifts of curves* that you have seen before.

0A.2.1 The Demand Curve A *demand curve* typically relates the quantity x demanded by a consumer to the price p_x of the good in question. It is therefore a two-dimensional graph, with

GRAPH 0.7 Movements along and Shifts in Demand Curves



x on the horizontal axis and p_x on the vertical. And when we graph such a demand curve, we are necessarily *holding all else fixed* and simply illustrating how the consumer's behavior changes as p_x changes.

Demand curves typically hold other variables fixed, like income. When these other variables change, the demand curve shifts.

In panel (a) of Graph 0.7, for instance, the magenta curve illustrates how the demand for x (on the horizontal axis) changes when the price of x (on the vertical axis) changes. As p_x increases, we slide up *along the demand curve,* and as p_x falls, we slide down *along the demand curve*.

But a consumer's decision about how much of a good x to buy does not usually depend only on the price p_x. The consumer's income I, for instance, might also play a role in determining how the consumer responds to price changes. When we plot a demand curve in panel (a), we therefore assume that I is fixed—which is equivalent to assuming that we are operating on a "slice" of a three-dimensional object that relates x to both p_x and I. This implies that when income changes, we move to a different "slice" of the underlying three-dimensional object—which is shown as a *shift in the demand curve* as illustrated in panel (b) of the graph.

In fact, a consumer's demand for x generally depends not only on p_x and I but also on the price of other goods. Were we to graph the three-dimensional demand function that relates x to p_x and I, this three-dimensional graph would then itself be a "slice" of a higher dimensional object that relates x also to other prices—and those other prices are held fixed when we graph the three-dimensional demand function just as they are in the two-dimensional graph of panel (a). And again, as prices of other goods change, the demand curve itself will shift as indicated in panel (b).

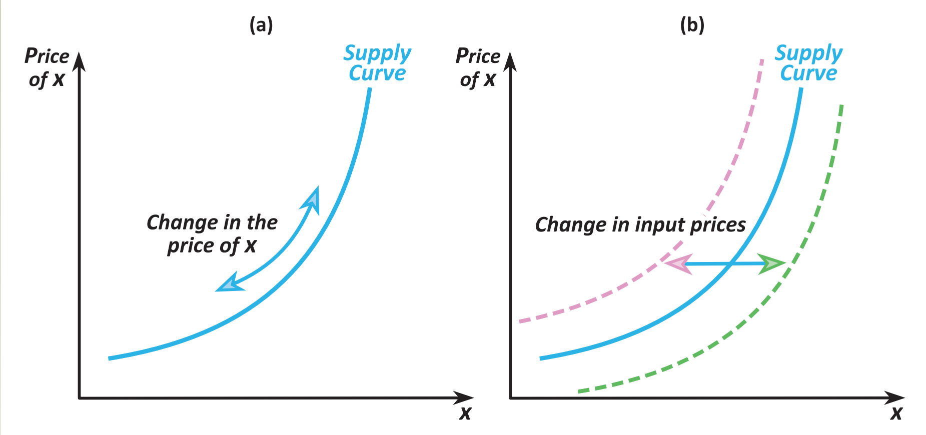
EXERCISE 0A.11 Suppose I like eating steak and will eat more steak as my income goes up. Which way will my demand curve for steak shift as my income increases? Can you think of any goods for which my demand curve might shift in the other direction as my income increases?

EXERCISE 0A.12 Coffee and sugar are *complements* for me in the sense that I use sugar in my coffee. Can you guess which way my demand curve for coffee will shift as the price of sugar increases?

EXERCISE 0A.13 Ice tea and coffee are *substitutes* for me in the sense that I like both of them but will only drink a certain total amount of liquids. Can you guess which way my demand curve for coffee will shift as the price of iced tea increases?

0A.2.2 The Supply Curve Supply curves are similar to demand curves in that they relate a *quantity* on the horizontal axis to a *price* on the vertical. But now the interpretation of the

GRAPH 0.8 Movements along and Shifts in Supply Curves



curve is one of supply rather than demand —that is, the curve illustrates how the quantity of x that is supplied changes as the price of x changes, *all else being held constant*. As the price of x increases, we therefore move up *along the supply curve* to determine the quantity of x that is supplied—as illustrated in panel (a) of Graph 0.8.

But again, the quantity that is supplied by a firm (or by the market) will depend on other factors—factors such as the input prices (like wages) that firms have to pay in order to produce. As these other factors change, the supply curve will *shift*. Put differently, the full description of the quantity supplied is a function of factors other than p_x— implying a multidimensional object from which the supply curve is drawn as a "slice" that holds these other factors fixed. And when one of those other factors changes, we switch to a different "slice" of the underlying relationship, which then appears as a shift in the supply curve (as illustrated in panel (b) of the Graph).

Supply curves typically hold input prices fixed. When input prices change, supply curves shift.

EXERCISE 0A.14 How would the supply curve for a firm shift if the general wage rate in the economy increases?

EXERCISE 0A.15 Would you expect the supply curve for a firm that produces *x* to shift when the price of some other good *y* (that is not used in the production of *x*) increases?

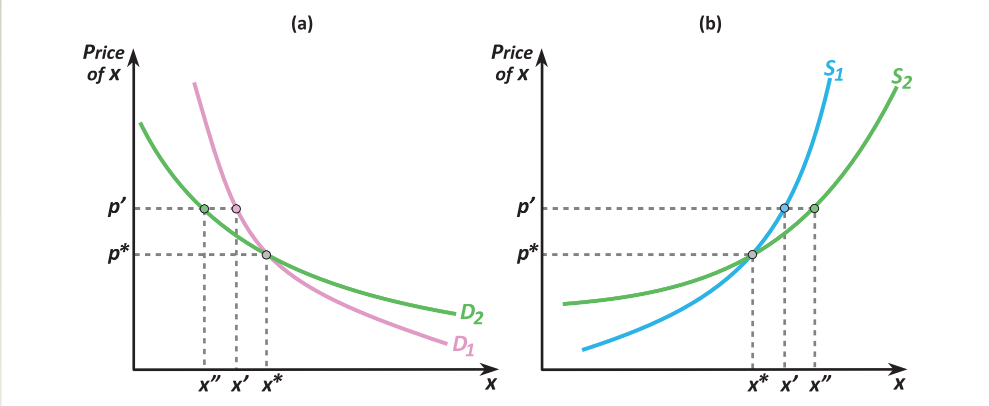
EXERCISE 0A.16 If the supply curve depicts the supply curve for a *market* composed of many firms, we may also see shifts in the supply curve that arise from the *entry* of new firms or the exit of existing firms. How would the market supply curve shift as firms enter and exit?

0A.2.3 Price Elasticities of Demand and Supply In many applications, we will consider how responsive consumers and producers are to price changes. Economists describe the responsiveness of behavior to prices by appealing to a concept known as *price elasticities*. We will introduce this concept more formally in Chapter 18 as the percentage change in quantity that arises from a one percent change in price. For now, we simply emphasize that elasticities capture *responsiveness* in behavior.

Elasticities capture responsiveness of behavior to changes in economic variables like prices.

Consider for instance two consumers who currently consume a quantity x^* at price p^*. Observing such consumption tells us one point on each of the consumers' demand curve—and since they are consuming the same amount at the same price, we know that this same point lies

GRAPH 0.9 Price Elasticities



on both consumers' demand curves. But without observing more choices at different prices, we can't tell whether the two consumers in fact share the same demand curve or whether their demand curves just happen to intersect at the quantity x^* and price p^*.

In panel (a) of Graph 0.9, we illustrate two demand curves consistent with observing the two consumers making the same consumption decision at price p^*. But the magenta demand curve for consumer 1 is steeper than the green demand curve for consumer 2 at the point at which the two curves intersect. This implies that, when price rises from p^* to p', the green consumer 2 will *respond more* than the magenta consumer 1: In particular, the green consumer reduces her consumption to x'' while the magenta consumer only reduces it to x'. We would then say that consumer 2's response to a price increase is *more elastic* (i.e., more responsive) than consumer 1's.

EXERCISE 0A.17 Is consumer 2's consumption also more elastic than consumer 1's when price falls?

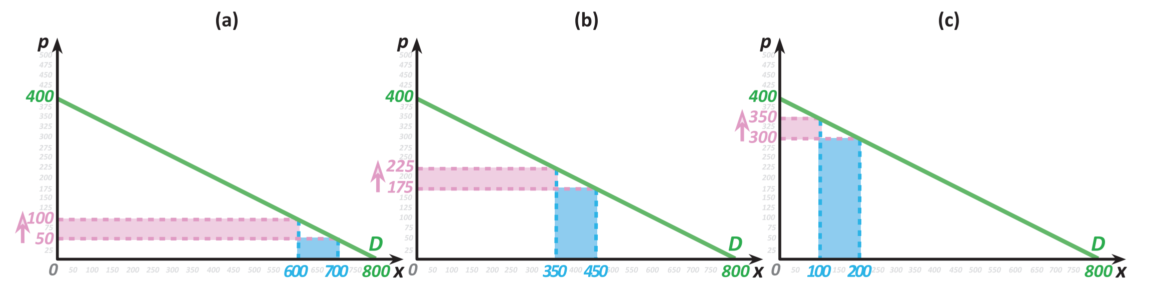
At the point at which two demand curves intersect, it is then pretty easy to determine whose behavior is more price *elastic*—it must be the person whose demand curve is shallower. As we will see in Chapter 18, however, it is misleading to use slopes of demand curves as the measure of price elasticity. This is in part because slopes can change if we simply measure output differently (i.e., kilos instead of pounds of rice; cans of soda versus liters of soda; etc.), but a measure of consumer responsiveness should not depend on how we measure output.

EXERCISE 0A.18 Do slopes similarly change if we measure price differently—that is, if we measure price in euros instead of dollars?

EXERCISE 0A.19 In panel (b) of Graph 0.9, the supply curves of two producers who both produce x^* at the price p^* p^* are illustrated. Which producer is more price elastic when price increases? What about when price decreases?

Another way to see the need for a unit-free measure of consumer responsiveness to price is to think of "responsiveness to price changes" in terms of what happens to consumer spending. When price increases, a consumer might spend less because she buys fewer goods, but she might also spend more because the remaining goods she does buy are more expensive. If the consumer is relatively unresponsive to price increases, we would expect the latter to outweigh the former—and consumer spending would increase as price rises. If, on the other

GRAPH 0.10 Spending and Price Elasticities



hand, the consumer is relatively responsive to price changes, we would expect the reverse to be true.

You can see this in the three panels of Graph 0.10,a graph that will appear again in Chapter 18. Here we have a linear demand curve and we consider how a consumer's spending changes as price increases from \$50 to \$100 in panel (a), from \$175 to \$225 in panel (b) and from \$300 to \$350 in panel (c). The shaded blue area in each panel is the *reduction in spending* from the fact that the consumer buys less at the higher price; the shaded magenta area in each panel is the *increase in spending* from the fact that the goods that are still bought at the higher price now cost more. Whenever the magenta area is larger than the blue area, a consumer's spending increases with a price increase; and whenever the blue area is larger than the magenta area, a consumer's spending decreases with a price increase.

Slopes of demand and supply curves are not generally a reliable way to capture respon-siveness.

EXERCISE 0A.20 How much does the consumer spend when price is \$50? How much does she spend when price increases to \$100?

EXERCISE 0A.21 What is the size of the blue shaded area in panel (a)? What about the magenta area? Is the difference between the magenta and the blue area the same as the increase in spending you calculated in exercise 0A.20?

If the slope of the demand curve were a good measure of price responsiveness, we would expect the same slope to imply the same impact on consumer spending for a similarly sized price increase. But Graph 0.10 clearly illustrates that, for a demand curve with the same slope throughout, consumer spending might respond positively or negatively to an increase in price.

0A.3 The Concept of Equilibrium

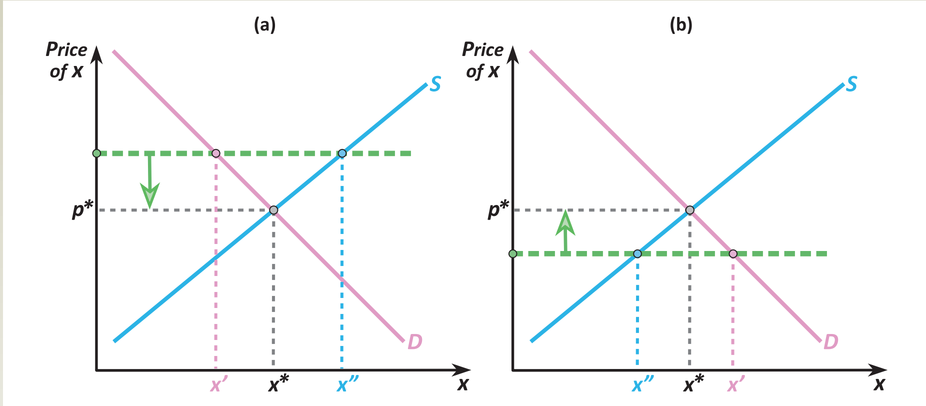
An *equilibrium* is a state of the world in which everyone is doing the best he can given how everyone else is behaving—that is, given what everyone is doing, no one wants to do anything differently. In many circumstances, this requires us to think carefully about not only how everyone currently behaves but also how that might change if some player changes his behavior. Such *strategic thinking* about how our actions interact with those of others is usually treated with the tools of *game theory,* a topic introduced in Chapter 24. But game theory is not always necessary to think about equilibrium, as you may already know from previous economics courses.

In an equilibrium, everyone is doing the best he can, given what everyone else is doing.

0A.3.1 Competitive Equilibrium

The most important case in which we do not have to think about game theory when thinking about equilibrium arises in *competitive settings*. We use the term

GRAPH 0.11 Competitive Equilibrium



competitive to describe "large" economic settings—settings in which each individual is sufficiently "small" relative to the economic environment such that nothing a single individual can do will impact how anyone else behaves. In such a case, there is no need for an individual to "think strategically"—that is, to think about how others might behave differently if he himself behaves differently.

In any previous economics course, for instance, you will almost certainly have graphed *market demand* and *supply curves* in the same picture. In that picture, you imagined many consumers whose actions together form the market demand curve, and many producers whose actions together form the market supply curve. And because there are many consumers and many producers, no single producer or consumer has any power to shift the market demand or supply curve.

When price in a competitive market deviates from the 'equilibrium price' at the intersection of demand and supply, producers and/or consumers have an incentive to change behavior in ways that drive price to the equilibrium price.

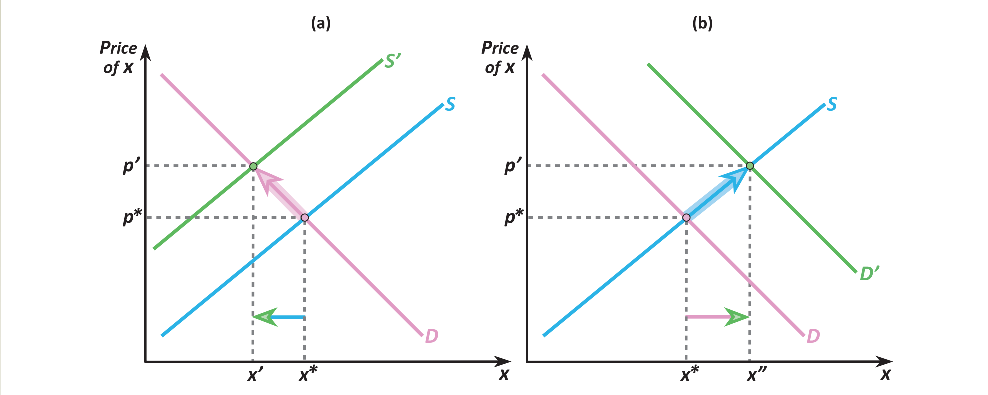
The two panels of Graph 0.11, for instance, illustrate magenta market demand curves and blue market supply curves. These curves intersect at the quantity x^* and the price p^*, an intersection that you will probably recall as the *equilibrium* in the graph—a quantity and price at which consumers and producers trade. The reason this intersection represents an equilibrium lies in the fact that, were the price ever to be anything other than p^* in this market, some individuals could do better by doing something different.

If the price occurred *above* p^* (as indicated by the green dashed line in panel (a) of the graph), consumers would only demand the magenta quantity x' while producers would want to sell the blue quantity x''. This implies that producers would not be able to sell all the goods they produce, which in turn implies that a single producer can do better by charging a price just below the green price, thus guaranteeing that he will sell his goods while everyone else charges the green price. Thus, as long as the price lies above p^*, individual firms will have an incentive to lower their price.

Similarly, if the price fell below p^* (as indicated in panel (b) of the graph), consumers would want to buy the magenta quantity x' that is now *larger* than the blue quantity x'' that producers are willing to sell at that price. Thus, a number of consumers will not be able to purchase the products they want at the green price. Firms might recognize lines of consumers outside their stores, or consumers might recognize that they can improve their chances of getting the products they want if they offered a higher price. Thus, as long as the price lies below p^*, individuals have an incentive to raise the price.

It is only when price is at p^* that every consumer who wants the product at that price can buy it, and every firm that is willing to sell at that price will in fact be able to sell it. A firm could offer a lower price—but that would mean it would do worse because it can already sell all the goods it can produce (given it is small relative to the market) at p^*. And any firm that

GRAPH 0.12 Increase in Equilibrium Price



tries to price above p^* will find all consumers going to its competitors. At the intersection point, everyone in the market is therefore doing the best he can.

A *price ceiling* is a government-enforced maximum legal price. In order for such a price ceiling to have an impact on the price at which goods are traded, would it have to be set above or below the equilibrium price p^*?

EXERCISE 0A.22

If a price ceiling changes the price at which goods are traded, would you expect a "shortage" or a "surplus" of goods to emerge? How would the magnitude of the shortage or surplus be related to the price elasticity of demand?

EXERCISE 0A.23

A *price floor* is a government-enforced minimum legal price. Repeat the previous two questions for a price floor instead of a price ceiling.

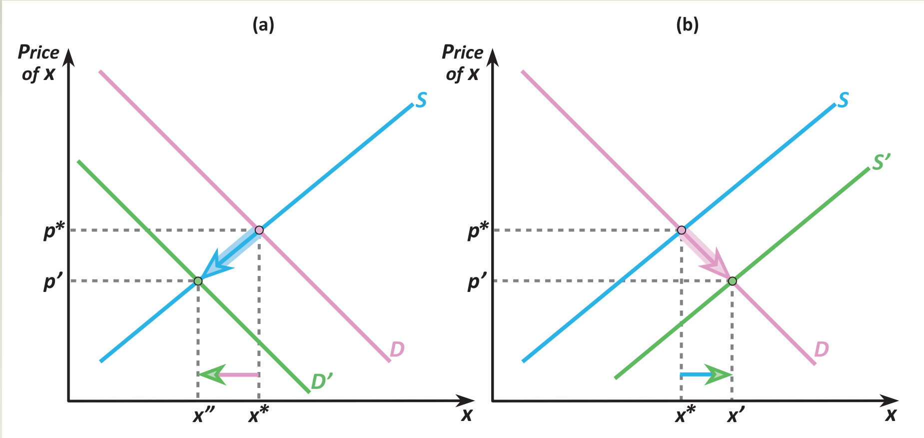
EXERCISE 0A.24

0A.3.2 Changes in Competitive Equilibria But real-world markets are often not in equilibrium for very long—things constantly change, shifting market demand and supply curves and thus putting upward or downward pressure on prices. An increase in labor costs, for instance, will cause an increase in costs for firms, and this in turn will imply that firms will require a higher price for any output they are willing to sell. As a result, the supply curve shifts "up" or "to the left" as depicted by the new green supply curve in panel (a) of Graph 0.12, with the new equilibrium now occurring at the intersection of the green supply curve with the magenta demand curve. Alternatively, perhaps a general increase in consumer income has consumers demand more of x at any given price, thus causing the demand curve to shift "to the right" or "up" as illustrated in panel (b) of Graph 0.12. The new equilibrium then occurs at the intersection of the new green demand curve with the blue supply curve.

When input prices change, market supply curves shift, giving rise to a new equilibrium price. When consumer income changes, market demand shifts, again causing a new equilibrium price to emerge.

Notice that in both cases, the equilibrium price increases from the initial p^* to the new p'. But in panel (a), this is accompanied by a reduction in output (from x^* to x') while in panel (b) it is accompanied by an increase in output (from x^* to x''). When the upward pressure on price comes from a shift in the supply curve, we slide up the demand curve (as indicated by the magenta arrow in panel (a))—which leads to the reduction in output as consumers are no longer willing to buy as much at the new higher price. But when the upward pressure on price comes

GRAPH 0.13 Decrease in Equilibrium Price



from a shift in demand, we slide up the supply curve (as indicated by the blue arrow in panel (b))—which leads to an increase in price that our consumers are willing to pay, and that induces our firms to produce the additional quantity.

By knowing what happens to *both* price and output in a market, we can then identify which curves must have shifted disproportionately more. If a price increase occurs alongside an increase in market output, it must be that the primary cause of the price increase lies on the demand side (as in panel (b)); but if the price increase occurs alongside a decline in market output, we must be looking at a supply-side phenomenon (as in panel (a)).

EXERCISE 0A.25 Can you come to similar conclusions about decreases in market prices by looking at Graph 0.13?

0A.3.3 Price Elasticities and Changes in Market Equilibrium

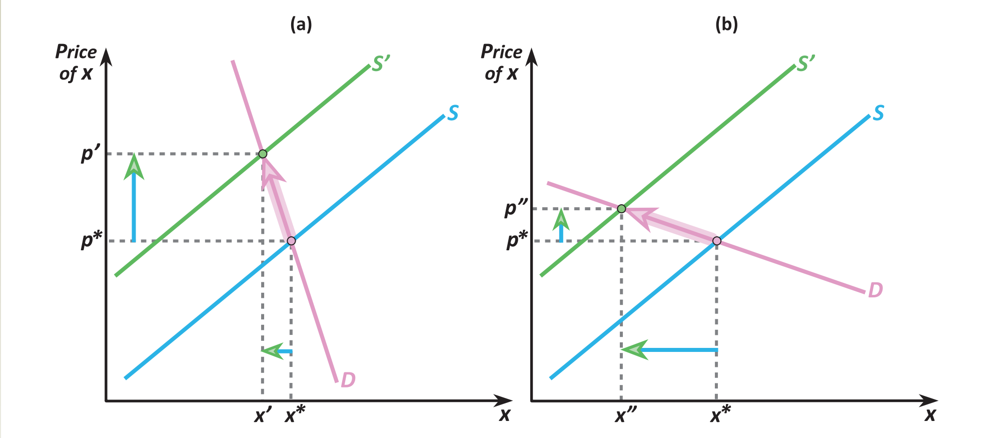
As we have seen, price increases arise from upward shifts in demand and/or supply curves—and the direction of accompanying quantity changes depends on which of the two curves is primarily responsible for the change in price. An understanding of the *direction* of market price and output changes then emerges from tracing through the impact of shifts in demand and supply curves. An understanding of the relative *magnitudes* of price and output changes, on the other hand, takes us back to price elasticities.

The more unresponsive behavior is to price, the greater the impact on price from shifts in supply and demand curves.

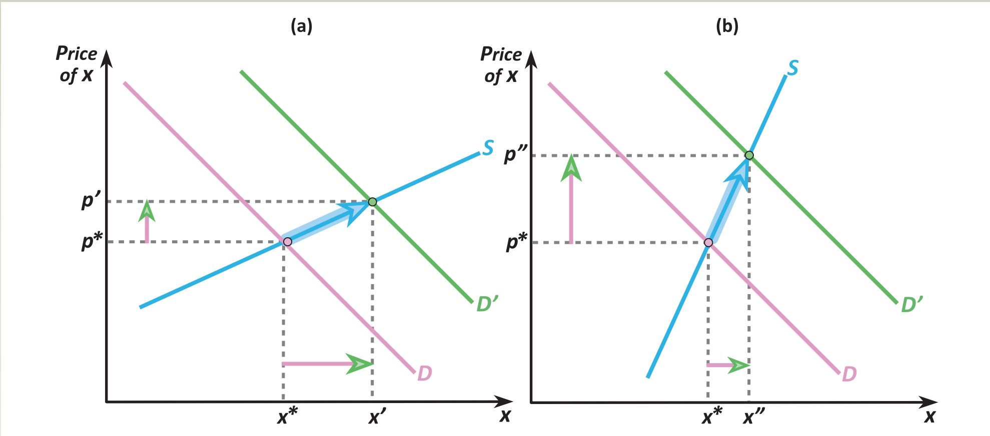
Graph 0.14, for instance, illustrates the same upward shift in the market supply curve for a relatively *inelastic* (or "unresponsive") demand curve in panel (a) and a relatively *elastic* (or "responsive") demand curve in panel (b). When consumers are relatively unresponsive to price changes (as in panel (a)), we see that the shift in supply results in a large increase in price but only a small reduction in output, whereas when consumers are relatively responsive (as in panel (b)), the reverse is true. Graph 0.15 illustrates the same idea for shifts in demand curves when supply is relatively price elastic (in panel (a)) and relatively price inelastic (in panel (b)). *Price inelasticities therefore lead to large market price changes relative to output changes as market curves shift.*

This has important implications for the impact of policies such as per-unit taxes. Suppose, for instance, that a per-unit tax is levied on producers of the x good. In other words, suppose that firms will have to pay an amount t for every unit of x they sell. This is, in effect, an increase in production costs of t per unit—which implies that a producer who was willing to produce at

GRAPH 0.14 Price Elasticity of Demand and Changes in Market Equilibria



GRAPH 0.15 Price Elasticity of Supply and Changes in Market Equilibria



price p will now only produce at price $(p + t)$. As a result, the market supply curve shifts up by t when the per-unit tax is imposed.

The fact that a tax is levied on firms, however, does not mean that firms will bear the full burden of the tax. Put differently, producers of x will seek to pass some of the burden onto consumers in the form of higher prices. But the degree to which producers are able to do this in equilibrium will depend on the price elasticity of demand. You can see this in Graph 0.14 where the same upward shift in supply leads to a large increase in price in panel (a) but only a small increase in price in panel (b). If this upward shift in supply is caused by a per-unit tax on firms, the firms will therefore be able to pass a significantly larger fraction of the tax onto consumers (in the form of higher prices) if consumers are relatively price-*inelastic*. This should, of course make intuitive sense: If consumers don't respond to price changes a lot, then it is easier to pass costs onto consumers without market output being dramatically impacted.[4]

The burden of a tax is more easily shifted onto consumers if consumer behavior is relatively unresponsive to price changes.

[4]In Chapter 19, we will see that it in fact does not matter whether the government imposes a tax on the consumer or the producer side of the market: the equilibrium distribution of the tax burden will depend entirely on the relative price elasticities of demand and supply.

In Graph 0.14, we can also see the impact that taxes have on economic activity. Since a per-unit tax on firms causes an upward shift of the supply curve, output will fall as a tax is increased. But output will fall significantly more as consumers become more price elastic in their behavior—with the reduction in output in panel (b) of Graph 0.14 being significantly larger than in panel (a).

EXERCSE 0A.26 Suppose that, instead of taxing the sale of x, the government subsidized consumer purchases of x. Thus, consumers will be paid an amount s for each good x they buy. Can you use Graph 0.15 to determine whether firms will benefit from such a consumer subsidy—and how the per-unit benefit for firms depends on the price elasticity of demand?

EXERCISE 0A.27 If the goal of consumer subsidies is to raise economic output in a market, will the government be more likely to succeed in markets with high or low price elasticities of supply?

0B SOME MATHEMATICAL PRELIMINARIES

Math intimidates many students—but my students also often tell me that, once they understand the underlying economics, the math isn't actually that bad. I think my students are right—while the B-parts of the text use calculus, there is relatively little beyond a first calculus course that's really necessary. If you know how to take a derivative, you are pretty much set in terms of the math you will need. The real key is to understand the economics enough to set up the math problems.

At the same time, I recognize from my own experience that solving math problems can get frustrating when careless mistakes slip in. Such mistakes can introduce hours of detective work as we try to find where we went wrong—not because the math is "hard," but just because we are not good at finding mistakes once we make them. For this reason, while the word "calculus" often intimidates, the real key to avoiding the frustrations of solving math problems often lies in a thorough comfort level with *pre-calculus* and basic *algebra* concepts. We forget how to work with exponents or simple systems of equations if we haven't done it in a while—and then we start making careless mistakes that trip us up. This section is therefore intended to provide some basic review of some of the most important pre-calculus and elementary calculus concepts.

0B.1 Functions and Sets Defined with Functions

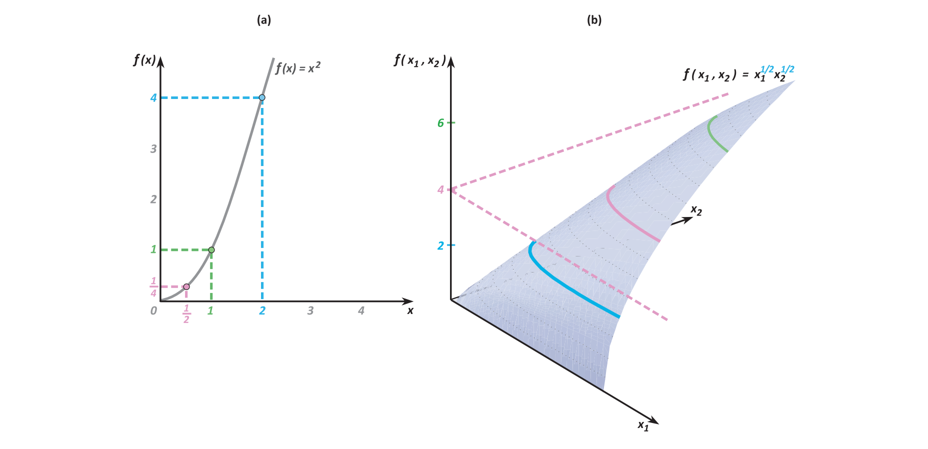
Functions are rules that assign values to points.

Functions are simply rules that assign values to points. As discussed in part A, when a point lies in one dimension, it is simply described by a value on the real number line, which we now denote as $\mathbb{R}^1$, or as $\mathbb{R}^1_+$ if we restrict ourselves to only positive real numbers. Similarly, a point that lies in two dimensions (such as the points in Graph 0.2) is described by a pair of numbers from two different real number lines—one pointing in the horizontal direction (the x-axis) and another pointing in the vertical direction (the y-axis). We will therefore denote such a point as lying in $\mathbb{R}^2$, or in $\mathbb{R}^2_+$ if we restrict ourselves to the positive quadrant (as we do in Graph 0.2). And a point that lies in N dimensions is analogously defined by N real numbers, each indicating the value it takes on each of the N dimensions. We will denote such a point as lying in the space $\mathbb{R}^N$ that is formed from N real number lines, or in $\mathbb{R}^N_+$ if we consider only positive values on each dimension.

A rule that assigns values to points in N dimensions can then be written as

$$f\colon \mathbb{R}^N \rightarrow \mathbb{R}^1. \qquad (0.5)$$

GRAPH 0.16 A Function from(a) $\mathbb{R}^1_+$ to $\mathbb{R}^1_+$ and from (b) $\mathbb{R}^2_+$ to $\mathbb{R}^1_+$



This notation is read as follows: The function f takes points that lie in the space $\mathbb{R}^N$ and assigns to these points a value from the real number line $\mathbb{R}^1$. Graph 0.16, for instance, plots the function $f(x) = x^2$ for $x > 0$—a function $f\colon \mathbb{R}^1_+ \rightarrow \mathbb{R}^1_+$; that is, a function that takes points from the (positive portion of the) one-dimensional x-axis and assigns to them a (positive) value $f(x)$. To the x-value 1, it assigns the value $f(1) = 1^2 = 1$, and to the x-value 2 it assigns the value $f(2) = 2^2 = 4$. The function $f(x_1, x_2) = x_1^{0.5}x_2^{0.5}$, on the other hand, is a function $f\colon \mathbb{R}^2_+ \rightarrow \mathbb{R}^1_+$ that takes points from the two-dimensional plane $\mathbb{R}^2_+$ and assigns a value from $\mathbb{R}^1_+$. To the two-dimensional point $(x_1, x_2) = (1, 1)$, for instance, it assigns the value $f(1, 1) = 1^{0.5}(1)^{0.5} = 1$, and to the point $(x_1, x_2) = (4, 4)$, it assigns the value $f(4, 4) = 4^{0.5}(4)^{0.5} = 2(2) = 4$. This function is graphed in panel (b) of Graph 0.16.

EXERCISE 0B.1

Consider the function $f(x,y,z) = xy + z$. How would you describe this function in terms of the notation of equation (0.5)? What value does the function assign to the points (0,1,2), (1,2,1), and (3,2,4)?

Functions such as these can then be used to define sets of points. For instance, the set of points that lie underneath the function graphed in panel (a) of Graph 0.16 can be described by the expression

$$\{(x, y) \in \mathbb{R}^2_+ \,|\, y \leq x^2\}. \tag{0.6}$$

This is read as "the set contains points in $\mathbb{R}^2_+$ for which the y component is less than or equal to the x component squared." The portion of the expression that precedes the vertical line | describes a *necessary condition* that points in the set must satisfy; that is, the points must lie in the positive quadrant of two-dimensional space. The portion of the expression that follows the vertical line |, on the other hand, describes the *sufficient condition* for points to lie within the set; that is, the relationship between the y component and the x component must be such that y is less than or equal to x^2.

EXERCISE 0B.2

How would you write the expression for the set of points that lie above the function in panel (a) of Graph 0.16? Which is different from expression (0.6): the necessary or the sufficient condition?

You can then similarly use *any* function to describe a set. The set of points that lie underneath the function described in panel (b) of Graph 0.16 is simply

$$\{(x_1, x_2, y) \in \mathbb{R}^3_+ \,|\, y \leq x_1^{1/2} x_2^{1/2}\}. \tag{0.7}$$

EXERCISE 0B.3 Is this set a convex set? What about the set described in expression (0.6) and the set defined in exercise 0B.2?

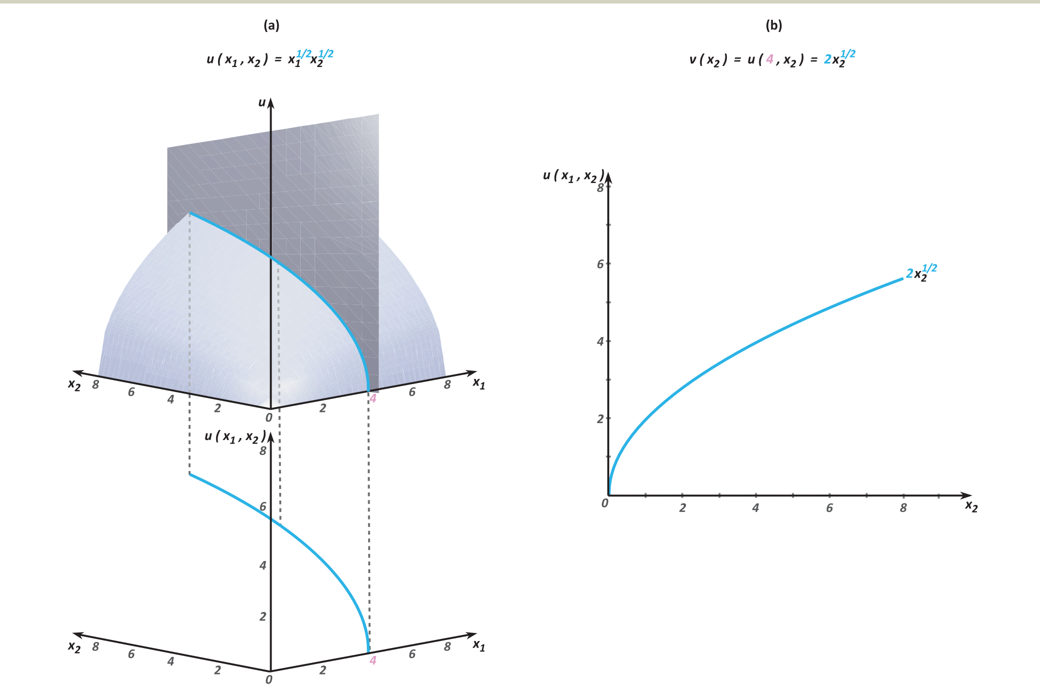
0B.1.1 Functions and "Slices" of Functions In part A, we described the fact that "slices" of three-dimensional graphs that hold one variable fixed can be represented in two dimensions—with changes in the fixed variable resulting in *shifts* of two-dimensional curves. The equation for the three-dimensional plane in Graph 0.6, for instance, is $4x + 2y + z = 40$, which reduces to $4x + 2y = 40$ when the variable z is fixed to 0, or to $4x + 2y = 30$ when z is fixed to 10 (resulting in the parallel shift of the line graphed in panel (b) of Graph 0.6).

The same logic, of course, applies to more complicated functions. In the top portion of panel (a) of Graph 0.17, for instance, we graph the function $u(x_1, x_2) = x_1^{0.5} x_2^{0.5}$—equivalent to the function $f(x_1, x_2)$ graphed in panel (b) of Graph 0.16 but now viewed from a different angle. If we fix x_1 to the value 4, this function then becomes

$$v(x_2) = u(4, x_2) = 4^{1/2} x_2^{1/2} = 2x_2^{1/2}, \tag{0.8}$$

a function that can be graphed in two dimensions as illustrated in panel (b) of Graph 0.17.

GRAPH 0.17 Demand *Functions* and Demand *Curves*



0B.1.2 Demand Functions and Demand Curves Suppose, for instance, that a consumer's demand for good x can be described by the function $f(p_x, I)$ where p_x denotes the price of the good and I denotes the consumer's income. This implies that the consumer's demand choice for the good x depends on two variables—price and income, with $f: \mathbb{R}^2_+ \rightarrow \mathbb{R}^1_+$. Perhaps the function takes the form $f(p_x, I) = (I/2) - 10p_x$. If income is 100 and the price of x is 2, this implies that the consumer will demand $f(2{,}100) = (100/2) - 10(2) = 50 - 20 = 30$. Put differently, the demand function assigns a value of 30 to the point $(p_x, I) = (2{,}100)$, but, more generally, the function gives us a way of describing the consumer's demand choice *for any pair of* p_x *and* I.

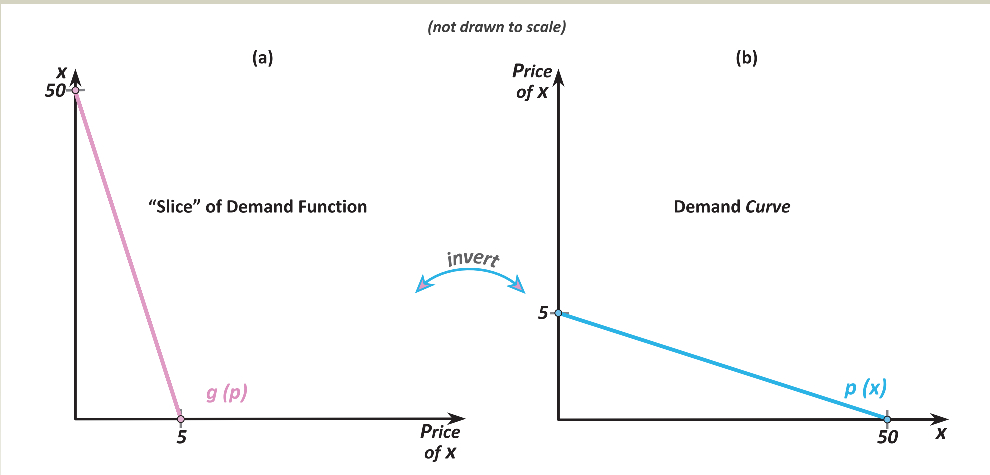
Demand curves are inverse slices of more complicated demand functions.

When you have graphed demand *curves* in previous classes, however, you probably only graphed the relationship of the consumer's demand choice with the price p_x. If the consumer's demand function is $f(p_x, I) = (I/2) - 10p_x$, you therefore implicitly held income I fixed and only considered the "slice" of the demand function with I held to a particular value. For instance, if you know the consumer's income is 100, we can write the slice of the demand function on which the consumer operates as $g(p_x) = f(p_x, 100) = (100/2) - 10p_x = 50 - 10p_x$. This slice of the demand function, graphed in panel (a) of Graph 0.18, is a function $g: \mathbb{R}^1_+ \rightarrow \mathbb{R}^1_+$—a function that assigns to every p_x value the number $g(p_x)$. And when you graphed the demand *curve*, you almost certainly ended up putting p_x on the vertical axis and x on the horizontal—implying that you did not graph $g(p_x)$ but rather its inverse. With $g(p_x) = 50 - 10p_x$, the inverse is derived by simply replacing $g(p_x)$ by x and solving for p_x—giving us the demand curve

$$p_x(x) = 5 - \frac{x}{10} \tag{0.9}$$

that is graphed in panel (b) of Graph 0.18. We thus write the demand curve as a function $p_x: \mathbb{R}^1_+ \rightarrow \mathbb{R}^1_+$, a function that assigns to every x-value a price p_x. We do this simply because it is a convention in economics to graph demand curves with quantity x on the horizontal and price p_x on the vertical axis—even though price for an individual consumer really is not a function of x but rather the quantity demanded x is a function of price. Were we to graph demand curves the way a mathematician would think about them, we would graph the $g(p_x)$ function—the function that treats the quantity consumed by the consumer as a function of the price. Put differently, when we graph demand *curves* we are really graphing *inverse slices of demand functions.*

GRAPH 0.18 Slicing $u(x_1, x_2)$ to get $v(x_2)$



EXERCISE 0B.4 Suppose that the quantity of the good x that is demanded is a function of not only p_x and I but also p_y, the price of some other good y. How would you express such a demand function in the notation of equation (0.5)?

EXERCISE 0B.5 Following on exercise 0B.4, suppose the demand function took the form $f(p_x, p_y, I) = (I/2) + p_y - 2p_x$. How much of x will the consumer demand if $I = 100$, $p_x = 20$ and $p_y = 10$?

EXERCISE 0B.6 Using the demand function from exercise 0B.5, derive the demand *curve* for when income is 100 and $p_y = 10$.

0B.1.3 Shifts in Demand Curves We have just seen that a demand curve is typically the inverse of a function $g : \mathbb{R}^1_+ \rightarrow \mathbb{R}^1_+$, a function that assigns the quantity of x demanded by a consumer for different prices p_x of the good. When the function $g(p_x)$ is derived from a demand function $f : \mathbb{R}^2_+ \rightarrow \mathbb{R}^1_+$ that depends not only on p_x but also on some other variable like income I, g is a "slice" of f that holds the other variable fixed at some value. In the previous section, for instance, we derived $g(p_x) = 50 - 10p_x$ from $f(p_x, I) = (I/2) - 10p_x$ by assuming that $I = 100$.

Now suppose that our consumer gets a new job and her income increases from 100 to 200. We can then derive the consumer's new demand function *given her income is 200* as $\overline{g}(p_x) = 100 - 10p_x$.

EXERCISE 0B.7 Verify the last sentence of the previous paragraph.

Notice that g and $\overline{g}$ differ only in the intercept term, with the intercept increasing from 50 to 100 but the slope remaining –10. The increase in income has therefore caused a parallel shift in the g function. The demand *curves* before and after the income change are the inverses of g and $\overline{g}$, which we can write as

$$p_x(x) = 5 - \frac{x}{10} \text{ and } \overline{p}_x(x) = 10 - \frac{x}{10}. \tag{0.10}$$

Again, we see the demand *curve* shifts up as a result of the increase in income—with the intercept term doubling but the slope term remaining the same.

The general point we have now made repeatedly is that *single-variable functions that are derived from multi-variable functions by holding some variables fixed are "slices" of the multivariable functions that shift as the fixed variables are changed*. This gives rise to a common distinction between "a change in the quantity demanded" when price rises and "a shift in demand" when something else (like income) changes. The former is a movement along a demand curve—and the latter is a shift in that curve.

EXERCISE 0B.8 On a graph with p_x and I on the lower axes and x on the vertical axis, can you graph $f(p_x,I) = (I/2) - 10p_x$? Where in your graph are the slices that hold I fixed at 100 and 200? How do these slices relate to one another when graphed on a two-dimensional graph with p_x on the horizontal and x on the vertical axis?

EXERCISE 0B.9 Return to exercise 0B.5 and suppose that $I = 100$ and $p_y = 10$. What is $g(p_x)$? How does it change when p_y increases to 20? How does this translate to a shift in the demand curve?

EXERCISE 0B.10

The same relationship between demand functions and demand curves exists for supply functions and supply curves. Suppose, for instance, that supply is a function of the wage rate ω and the output price p and is given by the supply function $f(\omega, p) = p - 5\omega$. Illustrate the "slice" of the supply function that holds ω fixed to 10, and then derive from it the supply curve. (Hint: You should get two graphs analogous to Graph 0.18.)

EXERCISE 0B.11

What happens in your two graphs from exercise 0B.10 when ω changes to 5?

0B.1.4 Parameters versus Variables

In essence, "slicing" multi-dimensional functions as we have done earlier simply involves changing some of the *variables* into *parameters*. The function $u(x_1, x_2) = x_1^{0.5}x_2^{0.5}$ in Graph 0.17 has two variables—x_1 and x_2. When "slicing" the function at $x_1 = 4$ to obtain $v(x_2) = 2x_2^{05}$, we are simply treating the x_1 variable as a parameter set equal to 4.

When we derive a slice of a function by holding a variable fixed, we treat that variable as a parameter.

More generally, we will often express parameters in the form of Greek letters—like α, β, and γ—to differentiate them from variables in our functions. You will, for instance, see functions like $u(x_1, x_2) = x_1^{\alpha}x_2^{\beta}$—where the exponents simply stand in for some value to be assigned later. Notationally we clarify what is a variable and what is a parameter by how we write the function, with $u(x_1, x_2)$ indicating that there are two variables called x_1 and x_2, and a function $f(x_1, x_2, y)$ indicating that there is an additional y variable. Put differently, variables represent the components of the points that the function assigns numbers to—and parameters simply alter what precise values are assigned to those points.

0B.1.5 Solving Systems of Equations: Equilibrium

In many of the applications developed in this text, we will need to solve systems of equations. The simplest application of this arises in the calculation of market output and prices when presented by supply and demand curves. Suppose, for instance, you are given the equation of a demand curve

$$p_d(x) = A - \alpha x \tag{0.11}$$

and a supply curve

$$P_s(x) = B + \beta x. \tag{0.12}$$

Note that these functions are written as functions of only x—implying that A, B, α, and β are parameters, with the capital letters giving the intercepts and the Greek letters giving the slopes (as depicted in Graph 0.19).

Solving for a competitive equilibrium in a market involves setting supply equal to demand and solving for the intersections point.

The equilibrium price p^* occurs at the intersection of these curves—with the price p_s read off the supply curve equal to the price p_d read off the demand curve. We can therefore set the demand and supply curves equal to one another; that is,

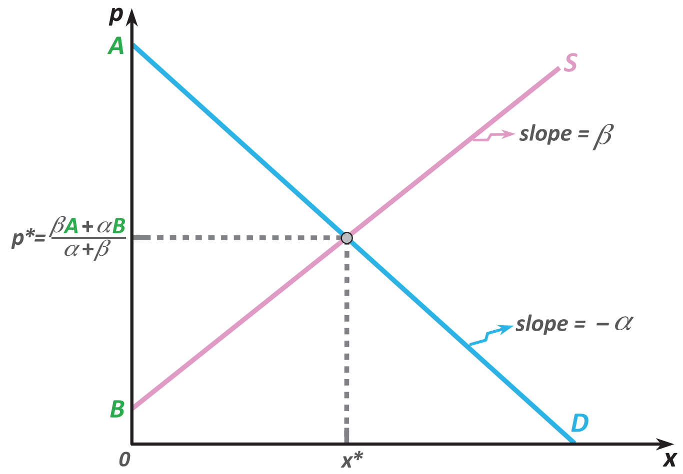
$$B + \beta x = A - \alpha x \tag{0.13}$$

and solve for x to get the market output level x^*. Subtracting B and adding αx to both sides, we get $(\alpha + \beta)x = A - B$, and dividing both sides by $(\alpha + \beta)$, we get the equilibrium quantity

$$x^* = \frac{A - B}{\alpha + \beta}. \tag{0.14}$$

As is obvious from Graph 0.19, both demand and supply curves pass through the same price p^* at the equilibrium output level x^*. We can therefore plug our expression for x^* from equation (0.14) into either the demand curve from equation (0.11) or the supply curve from equation (0.12). Using the demand curve, we get

GRAPH 0.19 Equilibrium as the Solution to a System of Equations



$$p^* = A - \alpha x^* = A - \alpha\left(\frac{A - B}{\alpha + \beta}\right) = \frac{\alpha A + \beta A - \alpha A + \alpha B}{\alpha + \beta} = \frac{\beta A + \alpha B}{\alpha + \beta}, \qquad (0.15)$$

and using the supply curve, we similarly get

$$p^* = B - \beta\left(\frac{A - B}{\alpha + \beta}\right) = \frac{\alpha\beta + \beta B + \beta A - \beta B}{\alpha + \beta} = \frac{\beta A + \alpha B}{\alpha + \beta}. \qquad (0.16)$$

EXERCISE 0B.12 Can you tell how equilibrium price changes as *A* and *B* change? Can you make intuitive sense of this by linking changes in parameters to changes in Graph 0.19?

EXERCISE 0B.13 Can you do the same for the equilibrium quantity?

Often, of course, the systems of equations we have to solve in an economic model are more complex—instead of 2 equations (supply and demand) and 2 unknowns (quantity and price), we might have 3 equations and 3 unknowns or 4 equations and 4 unknowns. As these systems become more complex, we tend to turn to increasingly sophisticated mathematical software packages that can do the work for us, but we'll frequently solve systems of at least 3 equations (and 3 unknowns) by hand.

There are multiple ways one can go about solving this. Consider a system of 3 variables (x, y, and z) and 3 equations. One method that's often easy to implement is to simply solve the first equation for x in terms of y and z, and then to substitute x in the second and third equations with the expression for x in terms of y and z. We then have a system of two equations and two unknowns (y and z) that we can solve for y and z in terms of just the parameters of the equations. Substituting those solutions back into the first equation then also gives us x in terms of just the parameters.

But often there are even simpler ways of solving some of the systems of equations we'll encounter. In Chapter 6, for instance, we encounter the system of equations

$$\frac{1}{2}x_1^{-1/2}x_2^{1/2} - 20\lambda = 0$$
$$\frac{1}{2}x_1^{1/2}x_2^{-1/2} - 10\lambda = 0 \qquad (0.17)$$
$$200 - 20x_1 - 10x_2 = 0$$

where x_1, x_2, and λ are variables.[5] Many of our systems of equations in the text will take this form, and we will solve these by re-writing the first two equations as

$$\frac{1}{2}x_1^{-1/2}x_2^{1/2} = 20\lambda$$
$$\frac{1}{2}x_1^{1/2}x_2^{-1/2} = 10\lambda \qquad (0.18)$$

and then dividing the equations by one another to cancel λ and get the equation

$$\frac{x_2}{x_1} = 2. \qquad (0.19)$$

This can then be written as $x_2 = 2x_1$ and substituted into the third equation (that lacks a λ variable) to give us

$$200 - 20x_1 - 10(2x_1) = 0 \qquad (0.20)$$

which then easily solves to give us $x_1 = 5$. Substituting this back into $x_2 = 2x_1$, we also get $x_2 = 10$.

In this Chapter 6 problem, it turns out we do not care about the value of λ. Suppose, however, we did. How would you derive it's value?

EXERCISE 0B.14

For most students, the process of solving the kinds of systems of equations we will encounter will quickly become relatively clear. However, in my experience, students often struggle with the nitty-gritty algebraic operations required to solve some of these—and so we turn to a brief review of some of these operations.

0B.2 Algebraic Operations

There are several algebraic concepts and techniques that appear frequently throughout the text. One involves solving systems of equations, a concept we already covered. Others involve working with exponents, quadratic equations and logarithms, each of which is discussed next.

0B.2.1 Rules for Working with Exponents A *positive integer exponent* simply indicates how many times we multiply the number (or variable) underneath the exponent. For instance, $2^3 = 2(2)(2) = 8$ and $x^4 = x(x)(x)(x)$. Anything with a zero exponent is equal to 1—that is, $2^0 = 1$ and $x^0 = 1$. A *negative exponent* indicates we are dividing rather than multiplying—that is, $2^{-3} = 1/(2^3) = 1/(2(2)(2)) = 1/8$ and $x^{-4} = 1/(x^4)$. Finally, a *fractional exponent*

[5]While we typically use Greek lower case letters to signify parameters rather than variables, this is not the case in the *Lagrange method* of optimization that is introduced in Chapter 6.

It is important to recall the basic rules of working with exponents.

implies we are taking a "root"—that is, $2^{1/2} = \sqrt{2}$ and $2^{1/3} = \sqrt[3]{2}$. Essentially everything you need to know about how to work with exponents follows from these four facts, but I am still amazed how often a careless mistake with exponents ends up costing me hours of backtracking when I solve the kinds of problems we will solve in the text.

It's useful, then, to keep the following rules for working with exponents in mind:

1. Anything taken to the exponent 0 is equal to 1: $x^0 = 1$.
2. Anything taken to the exponent 1 is equal to itself: $x^1 = x$.
3. Add exponents when multiplying: $x^m x^n = x^{(m+n)}$.
4. Subtract exponents when dividing: $\frac{x^m}{x^n} = x^{(m-n)}$.
5. Multiply exponents when you have an exponent on top of an exponent: $(x^m)^n = x^{mn}$.
6. Negative exponents imply division: $x^{-n} = \frac{1}{x^n}$.
7. Fractional exponents imply taking roots: $x^{1/n} = \sqrt[n]{x}$ and $x^{m/n} = (\sqrt[n]{x})^m$.
8. Keep track of exponents associated with different variables: $(xy)^n = x^n y^n$ and $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$.

EXERCISE 0B.15 Can you prove Rules 3 through 5 by just using the definition of exponents?

EXERCISE 0B.16 Simplify the following: $\frac{x^{3/2}y^{1/2}}{x^{1/2}y^{-1/2}}$.

EXERCISE 0B.17 Simplify the following: $\left(\frac{x^{4/5}y^2}{x^{-1/2}y^4}\right)^{-2}$.

0B.2.2 Quadratic Equations, Factoring, and the Quadratic Formula

A *quadratic equation* can be written in the form

$$ax^2 + bx + c = 0 \tag{0.21}$$

where a, b, and c are parameters. Such equations typically have two solutions, but in economic applications only one of the two solutions is typically economically meaningful.

When quadratic equations cannot easily be factored, the quadratic formula can be used to solve them.

Sometimes, the left-hand side of the quadratic equation can be easily *factored*—that is, it can be decomposed into a product of two objects (known as *factors*) that, when multiplied together, give us back the left-hand side of the equation. For instance, the left-hand side of $x^2 + 2x - 8 = 0$ can be factored as $(x + 4)(x - 2)$, with the two factors of $(x + 4)$ and $(x - 2)$ giving back the expression $x^2 + 2x - 8$ when multiplied out. The quadratic equation can then be written as $(x + 4)(x - 2) = 0$—which can hold only if at least one of the factors is equal to zero. Thus, we can conclude that there are two solutions to the quadratic equation: $x = -4$ and $x = 2$. In many economic applications, only positive solutions have economic meaning—which would then allow us to conclude that the latter is the economically meaningful solution.

EXERCISE 0B.18 Solve the following quadratic equation by factoring: $2x^2 + 2x - 12 = 0$.

EXERCISE 0B.19 An equation is still quadratic if $b = 0$. Solve the following quadratic equation by factoring: $x^2 - 4 = 0$.

EXERCISE 0B.20 Does the quadratic equation $x^2 + 4x + 4 = 0$ have two solutions?

It is often not, however, possible to factor a quadratic equation straightforwardly. The *quadratic formula* then gives an easy way to factor any quadratic equation—and thus to find the solutions to any quadratic equation. In particular, any quadratic equation can be written as

$$ax^2 + bx + c = a\left(x - \frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(x - \frac{-b - \sqrt{b^2 - 4ac}}{2a}\right), \quad (0.22)$$

with the terms in parentheses generated by the quadratic formula. This formula simply states that the solutions to a quadratic equation $ax^2 + bx + c = 0$ are always

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \frac{-b - \sqrt{b^2 - 4ac}}{2a}. \quad (0.23)$$

EXERCISE 0B.21

Use the quadratic formula to verify your answer to exercises 0B.18, 0B.19, and 0B.20.

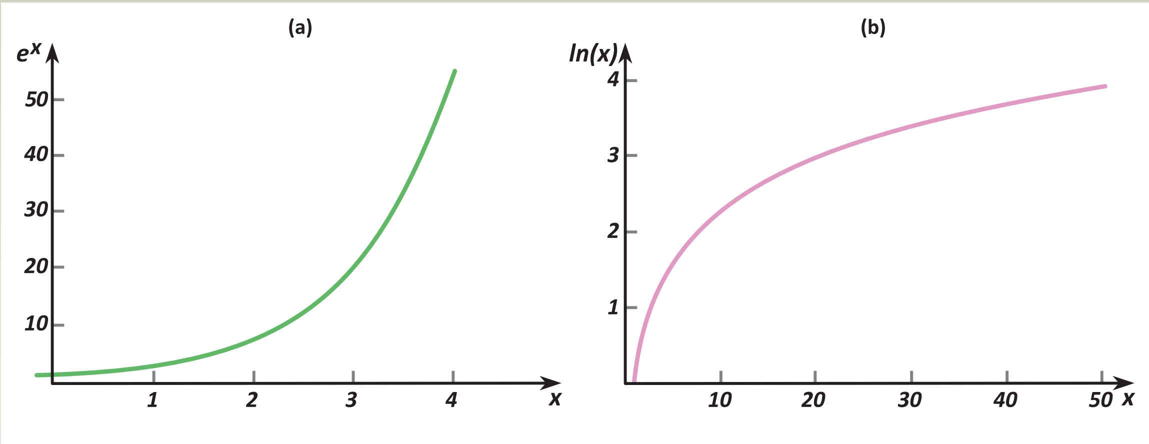
EXERCISE 0B.22

Solve the quadratic equation $3x^2 + 8x + 4 = 0$.

0B.2.3 Rules for Working With Natural Logarithms We often use logarithms in economic applications. The most common logarithm, and the one you probably first learned in your pre-calculus classes, is the *logarithm to the base 10.* This "base 10" logarithm of a number x, denoted $\log_{10} x$, is simply the exponent to which 10 would have to be raised to equal x; that is, since $10^2 = 100$, $\log_{10}100 = 2$.

Rather than using this "base 10" logarithm, however, we tend to instead use the *natural logarithm* defined as the logarithm *to the base e* (where e is an irrational constant approximately equal to 2.718).[6] The natural logarithm of a number x, usually denoted $\ln(x)$, is then the exponent to which e would have to be raised to equal x.

GRAPH 0.20 ln(x) in (b) is the Inverse of e^x in (a)



[6]Logarithms to any positive base vary only by a constant multiplier from the natural logarithm and can therefore be defined in terms of natural logarithms.

A related concept is that of an *exponential function* defined as $f(x) = e^x$, with the natural logarithm representing the *inverse of the exponential function*. This is illustrated in Graph 0.20 where panel (a) plots the exponential function e^x while panel (b) plots the (natural) logarithm function $\ln(x)$ (or $\log_e(x)$).

There are a few basic rules of logarithms that you'll have to keep in mind in applications that come up throughout the text. These are:

It is important to recall the basic rules of working with natural logarithms.

1. Logarithms turn multiplication into addition:

$$\ln(xy) = \ln(x) + \ln(y). \quad (0.24).$$

2. Logarithms turn division into subtraction:

$$\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y). \quad (0.25)$$

3. The inverse relationship between exponential functions and (natural) logarithmic functions imply

$$e^{\ln(x)} = x \text{ and } \ln(e^x) = x. \quad (0.26)$$

4. Logarithms are defined only for strictly positive values, with ln(0) undefined and In x approaching negative infinity as x approaches 0.

5. The natural log is positive for x-values that exceed 1 and negative for x-values that fall below 1; that is,

$$\ln 1 = 0, \ln x > 0 \text{ when } x > 1 \text{ and } \ln x < 0 \text{ when } 0 < x < 1. \quad (0.27)$$

EXERCISE 0B.23 Re-write the following as an expression with a single ln term: $\ln(2x) + \ln(y) - \ln(x)$.

EXERCISE 0B.24 Simplify the following: $\ln e^{(x^2+xy)}$.

EXERCISE 0B.25 Solve the following for x: $\ln e^{(x^2+4)} = \ln e^{4x}$.

0B.3 Some Basic Calculus

The textbook uses a single multi-variable concept heavily—the concept of *partial derivatives*. If you have taken a full calculus sequence prior to this course, you will have seen this concept already. If you have not, you will quickly see that it is a simple extension of single variable derivatives that you would have learned in your first calculus course. We leave the extension of derivatives to partial derivatives to the Appendix of Chapter 4 and review here only the single-variable calculus concepts you'll need.

Derivatives are slopes, and it is important to recall the basic rules of working with derivatives.

0B.3.1 Single-Variable Derivatives The derivative of a single-variable function $f(x)$ is simply the slope of that function. The first derivative of $f(x)$ is sometimes denoted as $f'(x)$ but more commonly as $\frac{df}{dx}$. Some frequently used rules for differentiating different types of functions include:

1. Derivatives of constants are equal to zero; that is, when γ is a constant, $\frac{d(\gamma)}{dx} = 0$.
2. Functions with constant slopes have derivatives equal to that constant; that is, $\frac{d(\gamma x)}{dx} = \gamma$.
3. Derivatives of higher-order polynomials follow the rule: $\frac{d(\gamma x^n)}{dx} = n\gamma x^{(n-1)}$.

4. The derivative of the sum of two functions is equal to the sum of the derivative of the functions; that is,

$$\frac{d(f(x) + g(x))}{dx} = \frac{df(x)}{dx} + \frac{dg(x)}{dx}. \tag{0.28}$$

5. The derivative of the natural log of x is the inverse of x: $\frac{d(\ln x)}{dx} = \frac{1}{x}$.
6. The derivative of the exponential function is the exponential function: $\frac{d(e^x)}{dx} = e^x$.
7. The derivative of the sin function is a cosine: $\frac{d(\sin x)}{dx} = \cos x$.
8. The derivative of the cos function is a negative sine: $\frac{d(\cos x)}{dx} = -\sin x$.

EXERCISE 0B.26

Show that Rule 2 follows from Rule 3.

EXERCISE 0B.27

Differentiate the following with respect to x: $f(x) = 3x^3 + 2x^2 + x + 4$.

Next, suppose you have a function $f(x)$ and another function $g(x)$—and suppose that these functions are multiplied, giving us $f(x)g(x)$. The derivative of this new function is then determined using the *product rule,* which states

$$\frac{d(f(x)g(x))}{dx} = \frac{df(x)}{dx}g(x) + f(x)\frac{dg(x)}{dx} \tag{0.29}$$

EXERCISE 0B.28

Use the product rule to differentiate the function $(x + 3)(2x - 2)$ with respect to x.

EXERCISE 0B.29

Multiply the function in exercise 0B.28 out and solve for its derivative with respect to x without using the product rule. Do you get the same answer?

Similarly, if a function is formed by dividing $f(x)$ and $g(x)$, the derivative of that new function can be evaluated using the *quotient rule:*

$$\frac{d\left(\frac{f(x)}{g(x)}\right)}{dx} = \frac{\frac{df(x)}{dx}g(x) - f(x)\frac{dg(x)}{dx}}{(g(x))^2} \tag{0.30}$$

EXERCISE 0B.30

Use the quotient rule to solve for the first derivative of $\frac{(x+3)}{2x-2}$.

Finally, the *chain rule* applies to instances where a function $g(x)$ becomes the argument in a function $f(x)$—that is, when we create the function $f(g(x))$ and differentiate it. For instance, suppose $f(x) = x^2$ and $g(x) = (x^3 - 2)$. Making $g(x)$ the argument in $f(x)$ implies $f(g(x)) = (x^3 - 2)^2$. The chain rule then tells us that

$$\frac{df(g(x))}{dx} = \frac{df(g)}{dg}\frac{dg(x)}{dx}. \tag{0.31}$$

Differentiating $f(g(x)) = (x^3 - 2)^2$ with respect to x then implies taking the derivative $\frac{df(g)}{g} = 2(x^3 - 2)$ as well as the derivative $\frac{dg(x)}{dx} = 3x^2$ to give us

$$\frac{df(g(x))}{dx} = 2(x^3 - 2)(3x^2) = 6x^5 - 12x^2. \tag{0.32}$$

EXERCISE 0B.31 Multiply out $(x^3 - 2)^2$ and take the derivative without using the chain rule. Do you get the same result?

EXERCISE 0B.32 In exercise 0B.30, you used the quotient rule to evaluate the derivative of $\frac{(x+3)}{2x-2}$. This function could alternatively be written as $(x + 3)(2x - 2)^{-1}$. Can you combine the product rule and the chain rule to solve again for the derivative with respect to x? Is your answer the same as in exercise 0B.30?[7]

EXERCISE 0B.33 Derive the derivative of $f(x) = \ln(x^2)$ using the chain rule. Then, use Rule 1 from our logarithm section to re-write $f(x)$ in such a way that you don't have to use the chain rule. Check whether you get the same answer.

0B.3.2 Example: Price Elasticities As we will see in Chapter 18, price elasticities of demand can be derived from demand functions for different points on the demand curve. You may know from a previous course, or you can see the idea developed in Section 18B.1, that the price elasticity of demand $\boldsymbol{\varepsilon}_d$ is defined as

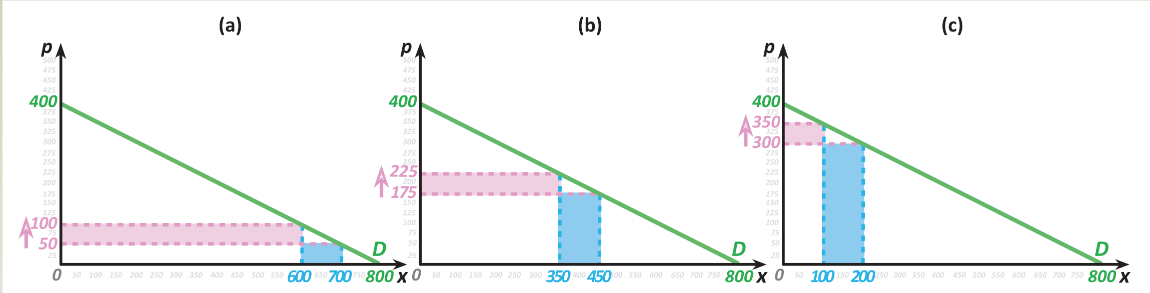
$$\varepsilon_d = \frac{dx(p)}{dp}\frac{p}{x(p)}. \tag{0.33}$$

In Graph 0.10 (replicated here as Graph 0.21), for instance, we graphed a demand *curve* $p(x) = 400 - 0.5x$—which is the inverse of the demand *function* $x(p) = 800–2p$. The derivative of this function with respect to p is $\frac{dx(p)}{dp} = -2$. Using the price elasticity formula, we can then derive the price elasticity of this demand curve at different prices as

$$\varepsilon_d = \frac{dx(p)}{dp}\frac{p}{x(p)} = -2\left(\frac{p}{800 - 2p}\right) = -\frac{p}{400 - p}. \tag{0.34}$$

To calculate the price elasticity at a particular point on the demand curve, we just have to plug in the price we are interested in. At a price of 75 (midway between the higher and lower

GRAPH 0.21 Spending and Price Elasticities



[7] This exercise illustrates that the quotient rule is in some way superfluous because we can always re-write a fraction in a form that allows us to apply the product rule with the chain rule.

price in panel (a) of the Graph), the formula gives us the price elasticity $-75/(400-75) \approx -0.23$. For the similar "in between price" of 200 in panel (b), on the other hand, the formula tells us that price elasticity is equal to $-200/(400-200) = -1$; and at the "in between price" of 325 in panel (c), we get the price elasticity of $-325/(400-325) \approx 4.33$.

Thus, the price elasticity is increasing in absolute value as we move up the demand curve. At low prices, a small price elasticity (in absolute value) indicates relatively little responsiveness of behavior to price—giving us the result that spending actually increases as price rises (because the magenta area is bigger than the blue area in panel (a)). At high prices, on the other hand, the large price elasticity (in absolute value) indicates relatively more responsiveness of behavior to price—with spending falling as price increases (as in panel (c) where the magenta area is now smaller than the blue area). At the midpoint of the demand curve (where price is 200), we get the price elasticity of –1 where changes in prices lead to roughly the same amount of spending.

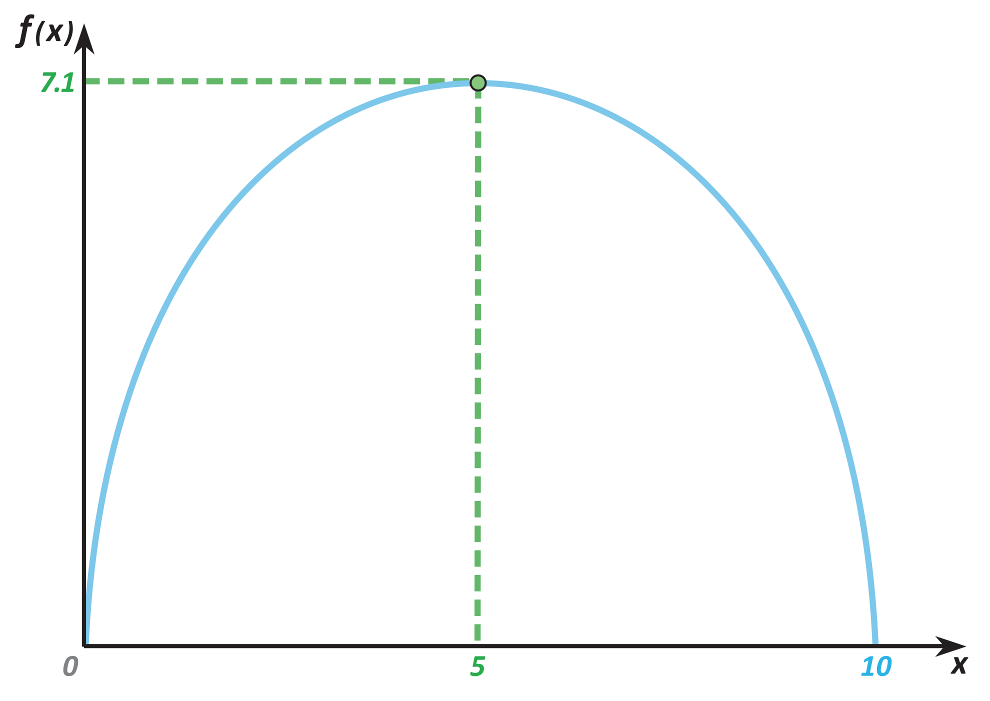
We have seen that price elasticities vary along linear demand curves. Consider now the demand curve given by the equation $x(p) = \alpha/p$. Is the same true for this demand function?

EXERCISE 0B.34

0B.3.3 Finding Maxima and Minima One of the main uses of calculus in economics emerges from *optimization problems*. Consumers do the best they can given their circumstances—that is, they "optimize" their well being subject to their constraints. Firms try to "optimize" profits. In fact, economics derives much of its identity from viewing human behavior as emerging from individuals solving optimization problems. In Chapter 6, we will begin to investigate the economics behind optimization. For now, all we hope to do is simply illustrate the usefulness of calculus in the process of optimizing single-variable mathematical functions.

Calculus is used to solve economic optimization problems.

GRAPH 0.22 Finding the Maximum of a Function $f(x) = x^{1/2}(20 - 2x)^{1/2}$



Consider, for instance, the function $f(x) = x^{1/2}(20 - 2x)^{1/2}$, which is illustrated in Graph 0.22 (and which takes on economic meaning in Chapter 6). From the graph, it is clear that the function attains its highest point at $x = 5$. It is also clear from the graph that the very definition of a "maximum" implies that the slope of the function must be zero at that maximum. Put differently, it has to be true that the derivative of the function is equal to zero at the point where the function attains its maximum.

The derivative of the function $f(x) = x^{1/2}(20 - 2x)^{1/2}$ is

$$\frac{df(x)}{dx} = \frac{1}{2}x^{-1/2}(20 - 2x)^{1/2} - x^{1/2}(20 - 2x)^{-1/2}. \quad (0.35)$$

EXERCISE 0B.35 Verify that this is correct.

Setting this derivative equal to zero implies that

$$\frac{1}{2}x^{-1/2}(20 - 2x)^{1/2} = x^{1/2}(20 - 2x)^{-1/2}. \quad (0.36)$$

Multiplying both sides by $x^{1/2}$ and by $(20 - 2x)^{1/2}$ gives us

$$x = \frac{20 - 2x}{2} = 10 - x. \quad (0.37)$$

Solving for x, we get $x = 5$—precisely where the maximum occurs in the graph.

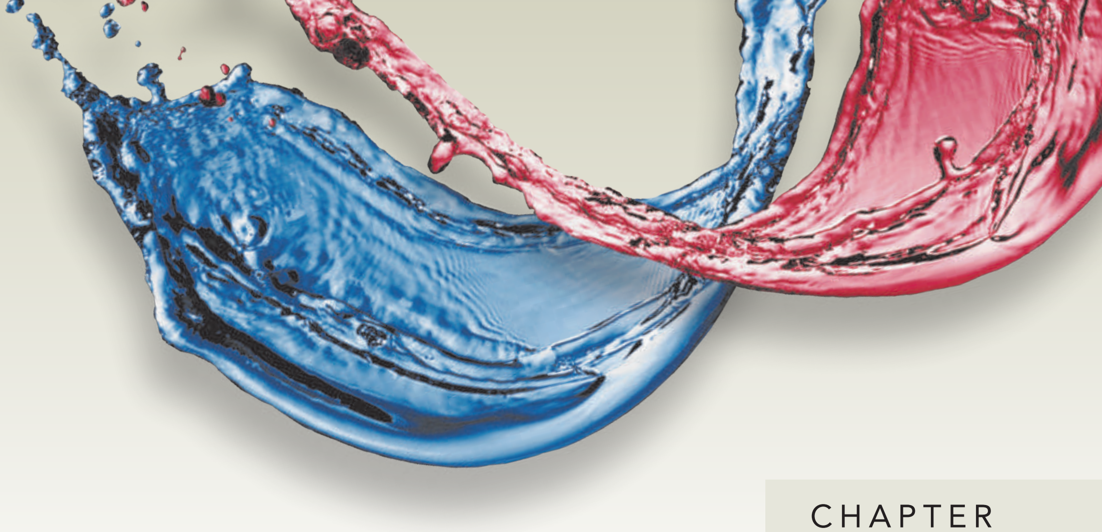
Of course it is not true that a derivative of zero implies a maximum for the function. A function with a U-shape (as opposed to the inverse U-shape in Graph 0.22) will have a zero slope (and thus a zero derivative) at its *minimum* rather than at a *maximum*. But it is true that the derivative of a function is equal to zero at any maximum—even if it is also true that the derivative of a function is zero at a minimum.

EXERCISE 0B.36 Evaluate the following statement: A derivative of zero is a necessary but not a sufficient condition for us to identify a maximum of a function.

0B.3.4 (Integrals) The concept of an integral is used only in a few sections of the text, and in each case you can easily skip those sections if you are not comfortable with the idea. In essence, an integral is simply a way of measuring areas under curves—something that can easily be done using basic rules of geometry when functions are linear.

CONCLUSION

This chapter began with an overview of some basic concepts used in the graphs that we will develop throughout the text, concepts such as *points, sets, convexity,* and the idea of lower-dimensional graphs as "slices" of higher dimensional graphs. The latter of these then gives us a way of thinking about the common Econ 1 distinction between "movements along curves" and "shifts of curves": When some variables are "fixed," the resulting "slices" we graph will move as the fixed variables are changed. Perhaps the most intuitive example is the difference between a price change moving us along a demand curve and an income change shifting the demand curve. In part B of the chapter we then formalized some of these ideas—and provided an overview of the most useful pre-calculus and single-variable calculus concepts used throughout the text.



Introduction

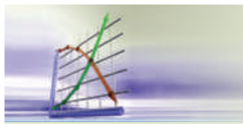
CHAPTER 1

Do safer cars necessarily result in fewer traffic deaths? Is it sensible to subsidize domestic U.S. oil drilling in an effort to make the United States less dependent on unstable regions of the world? Would outlawing live Christmas trees help to reduce deforestation? Should we impose laws against "price gouging"? Is boycotting companies that use cheap labor abroad a good way to express our outrage at the dismal working conditions in those countries? Would it be better for workers to require their employers to pay their Social Security taxes rather than taxing the workers directly? Should we tax the sales by monopolies so that these companies don't earn such outrageous profits?

Many people would instinctively answer "yes" to each of these questions. Many economists would say "no," or at least "not necessarily." Why is that?

One possible answer is that economists are social misfits who have different values than "real people." But I don't think that's typically the right answer. By and large, economists are an ideologically diverse group, distributed along the political spectrum much as the rest of the population. Most of us live perfectly normal lives, love our children, and empathize with the pain of others. Some of us even go to church. We do, however, look at the world through a somewhat different lens, a lens that presumes people respond to incentives and that these responses aggregate in ways that are often surprising, frequently humbling, and sometimes quite stunning. What we think we know isn't always so, and, as a result, our actions, particularly in the policy realm, often have "unintended" consequences.

I know many of you are taking this course with a hidden agenda of learning more about "business," and I certainly hope that you will not be disappointed. But the social science of economics in general, and microeconomics in particular, is about much more than that. Through the lens of this science, economists see many instances of remarkable social order emerging from millions of seemingly unconnected choices in the "marketplace," spontaneous cooperation among individuals on different ends of the globe, the kind of cooperation that propels societies out of the material poverty and despair that has characterized most of human history. At the same time, our lens clarifies when individual incentives run counter to the "common good," when private interests unravel social cooperation in the absence of corrective nonmarket institutions. Markets have given rise to enormous wealth, but we also have to come to terms with issues such as economic inequality and global warming, unscrupulous business practices, and racial discrimination. Economics can certainly help us think more clearly about business and everyday life. It can also, however, teach some very deep insights about the world in which we live, a world in which incentives matter.



Almost all graphs in the text can be viewed as narrated video animations within MindTap.